MESH ANALYSIS

A mesh is a loop which does not contain any other loops within it.

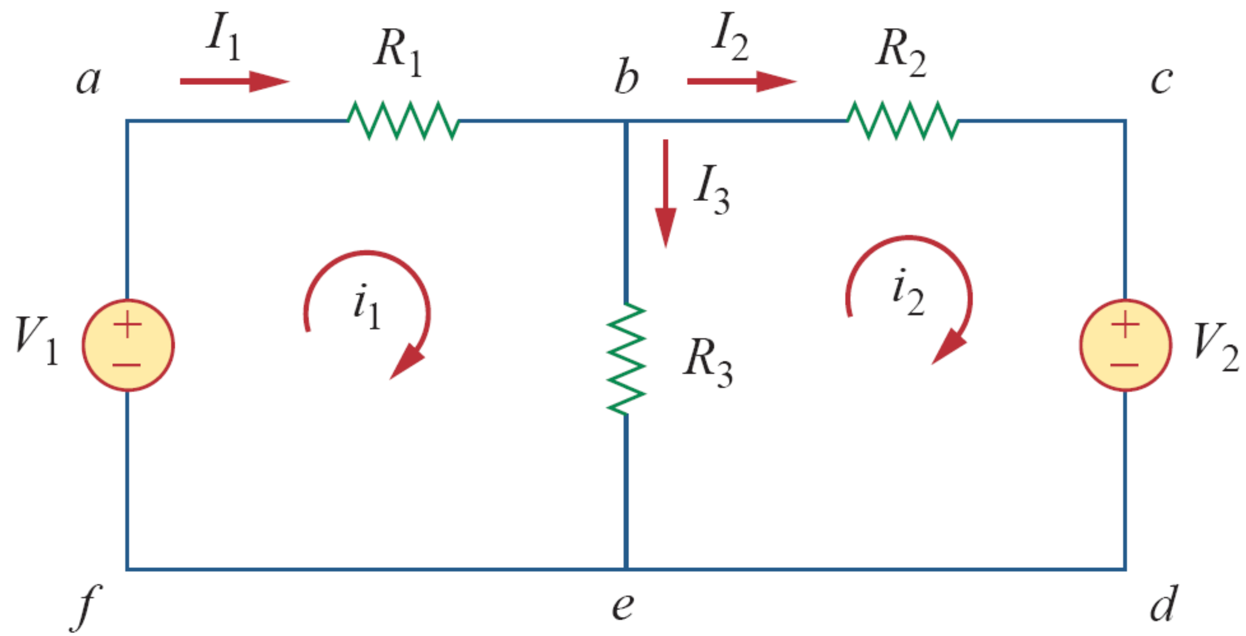

Figure 3.17

A circuit with two meshes.

Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

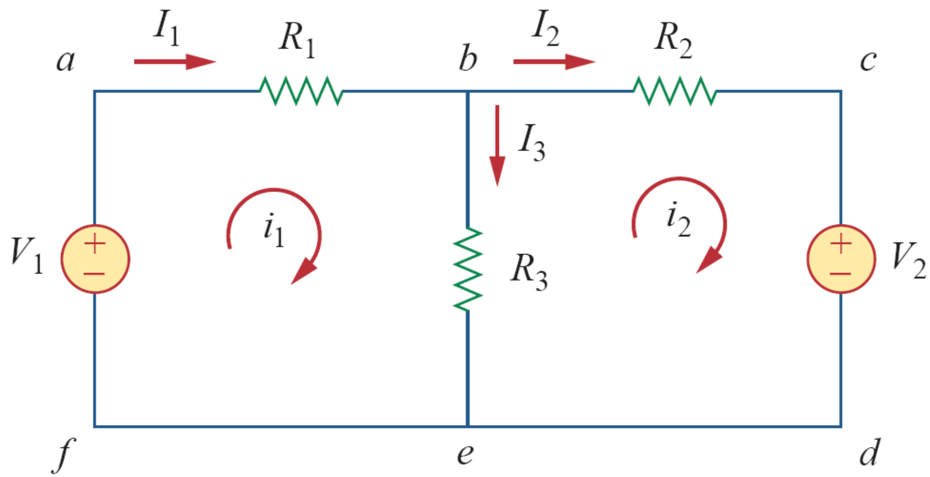

Figure 3.17
A circuit with two meshes.

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix} \tag{3.15}$$

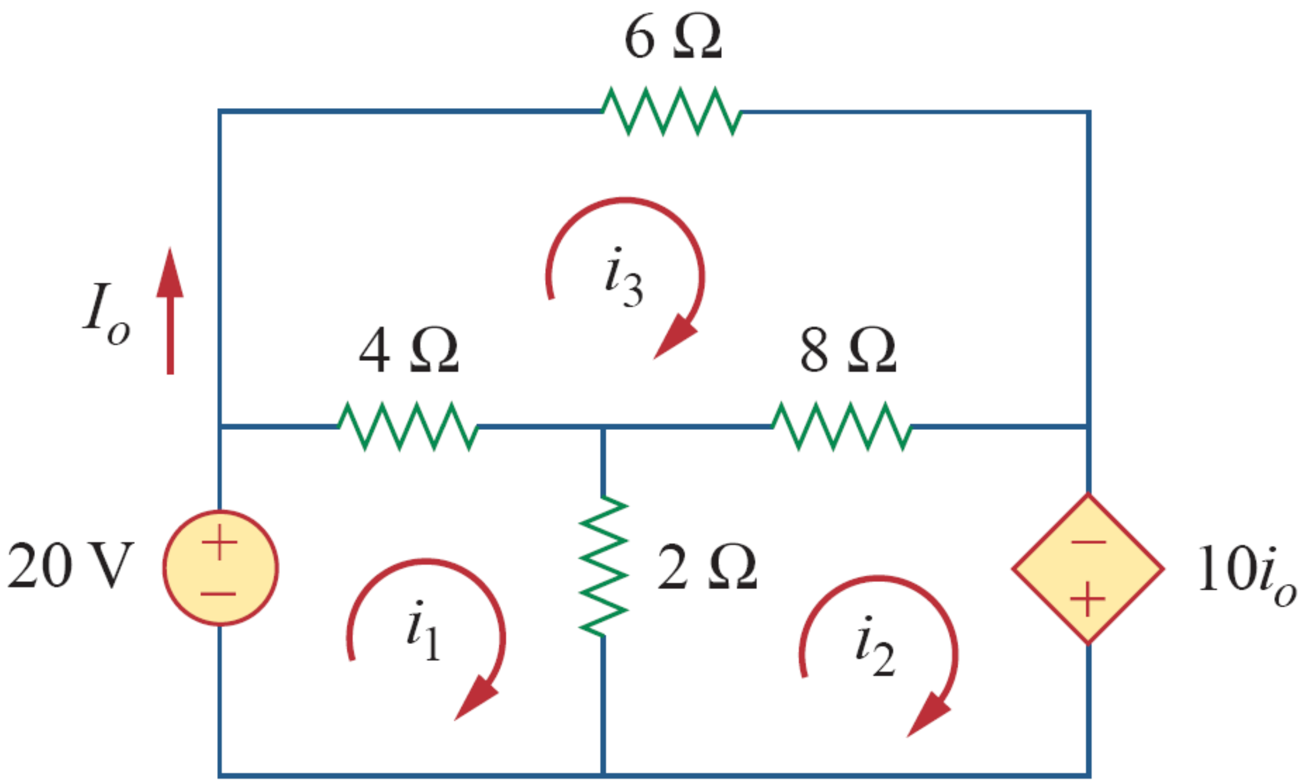

Figure 3.21

For Practice Prob. 3.6.

Two methods

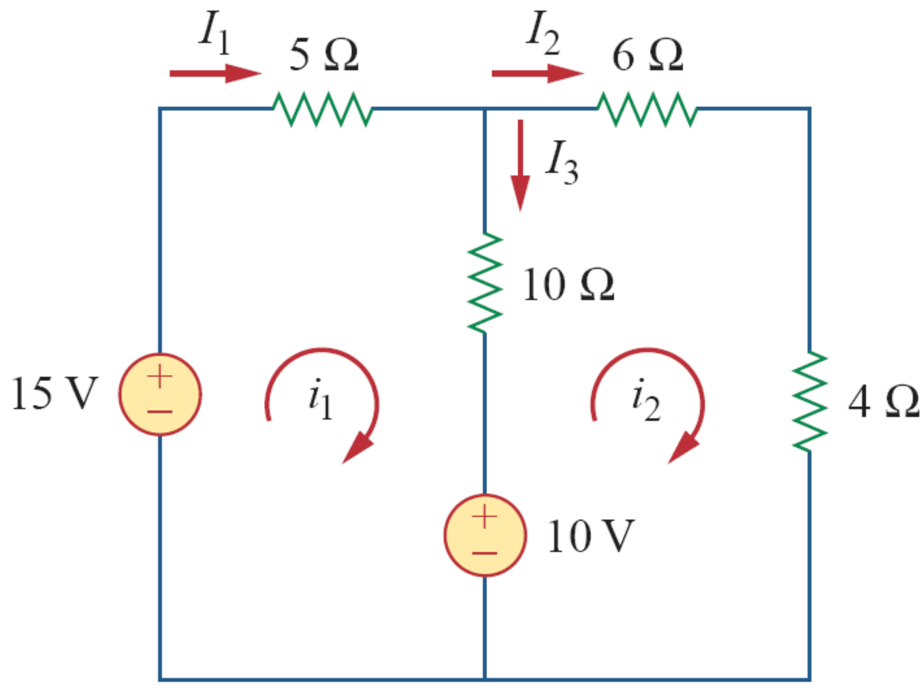

Figure 3.18
For Example 3.5.

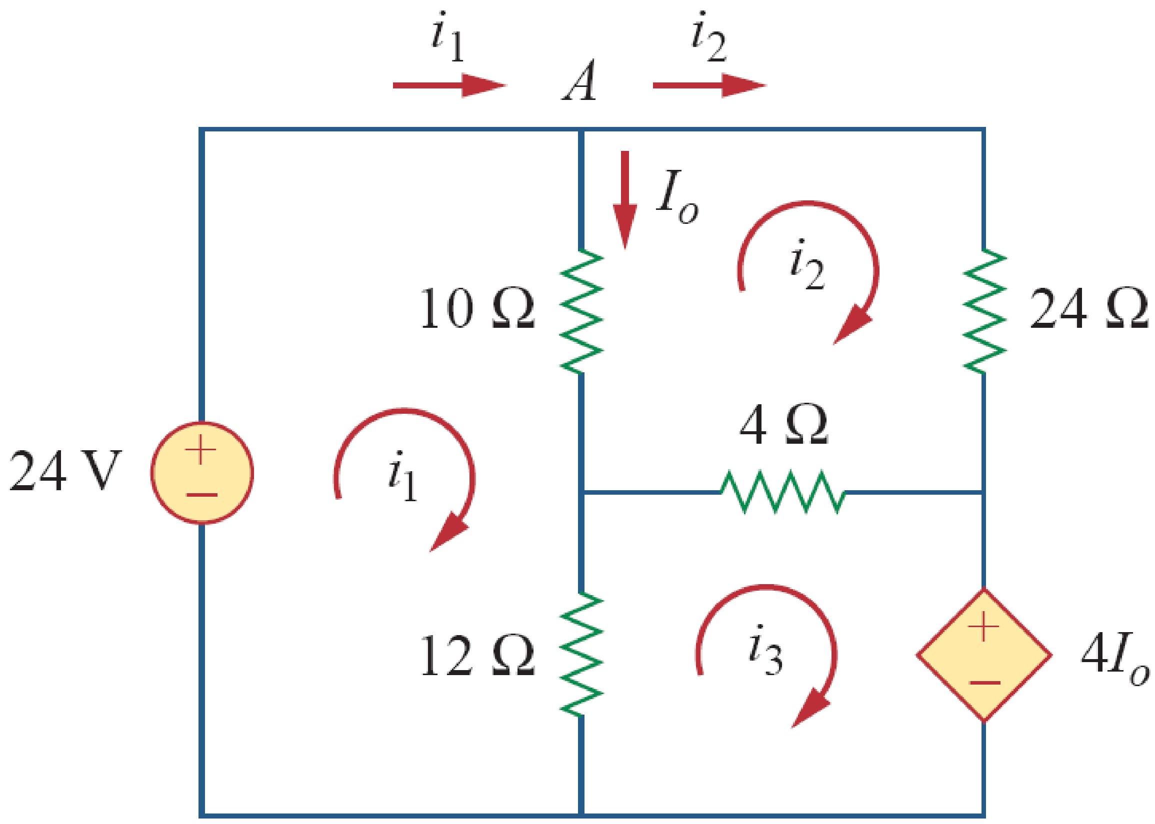

Figure 3.20
For Example 3.6.

Mesh Analysis with Current Sources

- **CASE 1 When a current source exists only in one mesh: Consider** the circuit in Fig. 3.22, for example.

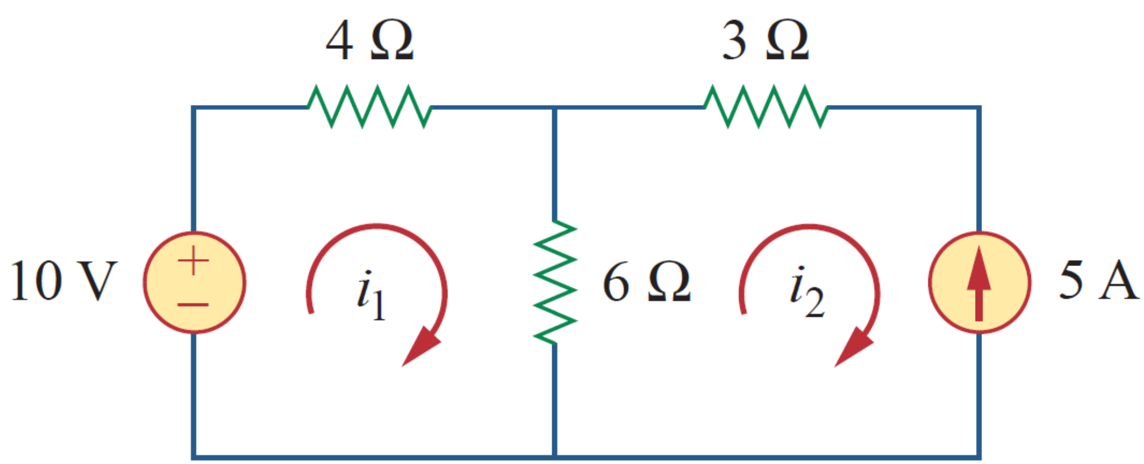

Figure 3.22
A circuit with a current source.

CASE 2 When a current source exists between two meshes:

- A supermesh results when two meshes have a (dependent or independent) current source in common.

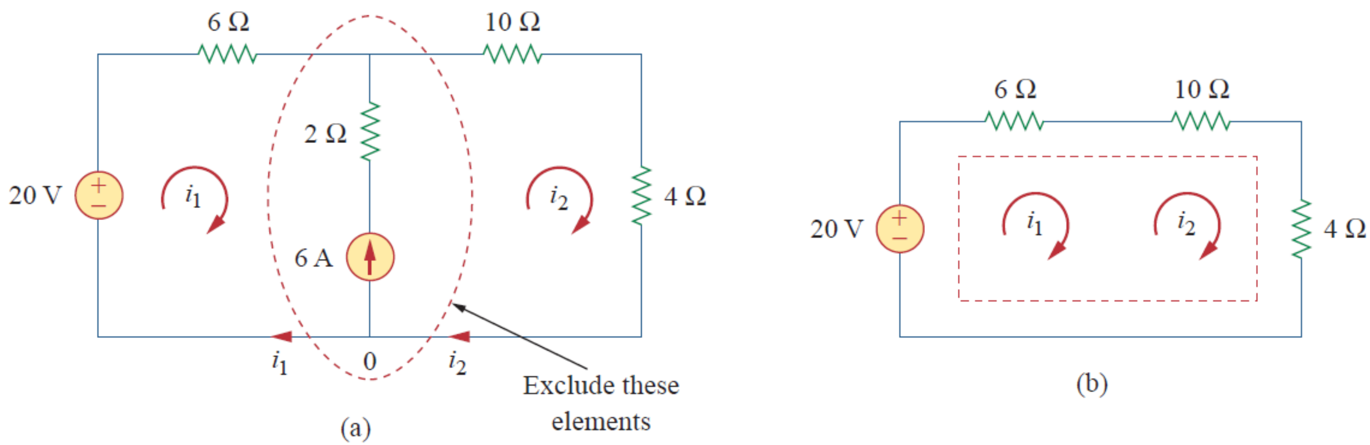

Figure 3.23
(a) Two meshes having a current source in common, (b) a supermesh, created by excluding the current source.

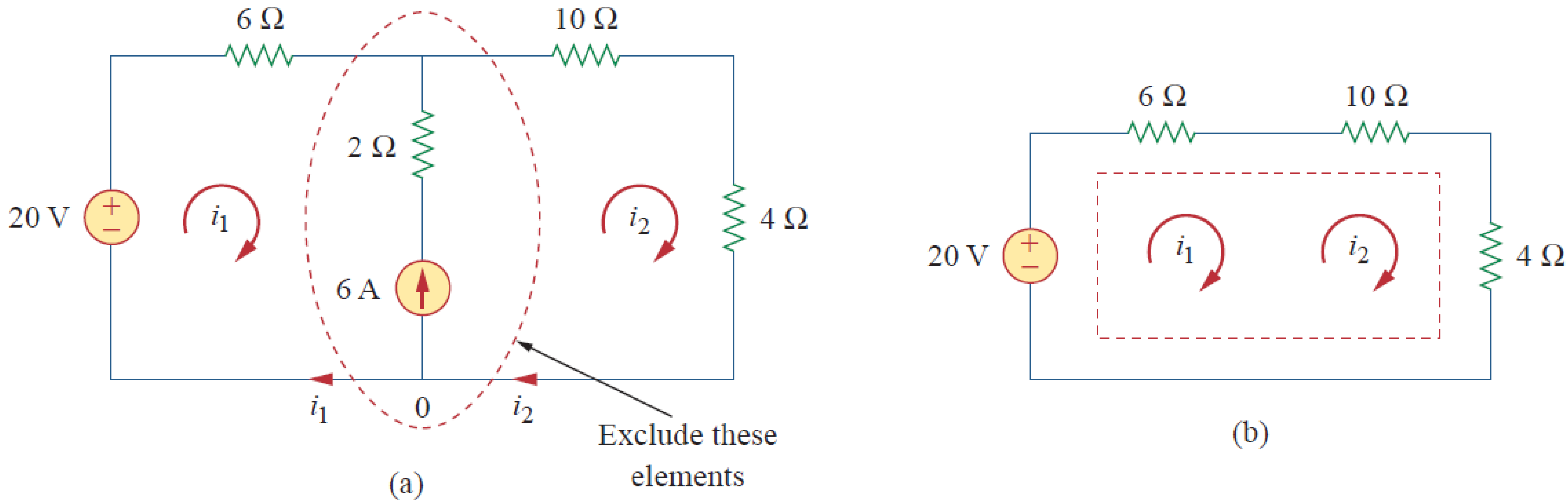

Figure 3.23
(a) Two meshes having a current source in common, (b) a supermesh, created by excluding the current source.

Why treat the supermesh differently? Because mesh analysis applies KVL—which requires that we know the voltage across each branch—and we do not know the voltage across a current source in advance.

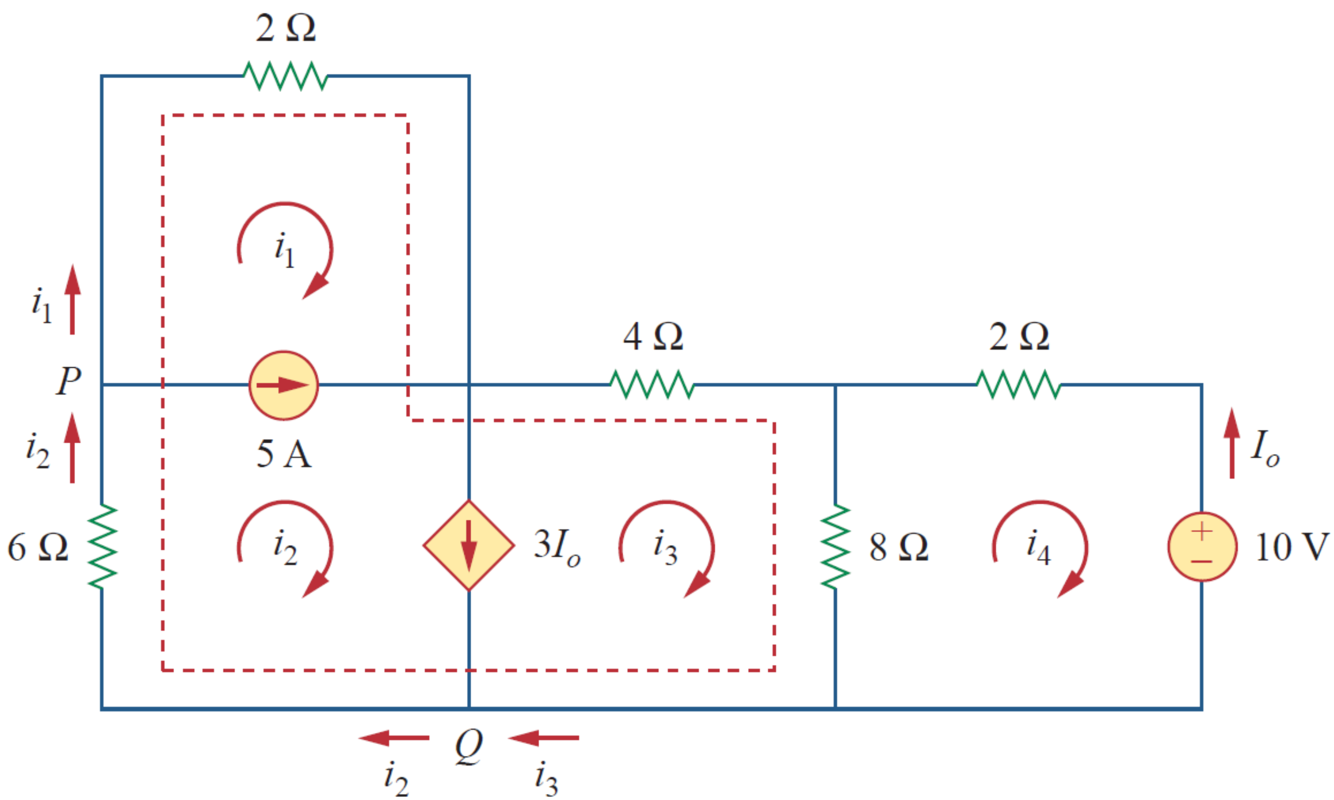

Figure 3.24
For Example 3.7.

Nodal and Mesh Analyses by Inspection

$$\begin{bmatrix} G_{11} & G_{12} & \dots & G_{1N} \\ G_{21} & G_{22} & \dots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \dots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} \qquad (3.22)$$

$$\mathbf{G}\mathbf{v} = \mathbf{i} \qquad (3.23)$$

where

G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j, $k \neq j$

v_k = Unknown voltage at node k

i_k = Sum of all independent current sources directly connected to node k, with currents entering the node treated as positive

G is called the *conductance matrix*; **v** is the output vector; and **i** is the

$$\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

$$\mathbf{Ri} = \mathbf{v}$$

R_{kk} = Sum of the resistances in mesh k

$R_{kj} = R_{jk}$ = Negative of the sum of the resistances in common with meshes k and j, $k \neq j$

i_k = Unknown mesh current for mesh k in the clockwise direction

v_k = Sum taken clockwise of all independent voltage sources in mesh k, with voltage rise treated as positive

is called the *resistance matrix*; **i** is the output vector; and **v** is input vector. We can solve Eq. (3.25) to obtain the unknown mesh rents.

Write the node-voltage matrix equations for the circuit in Fig. 3.27 by inspection.

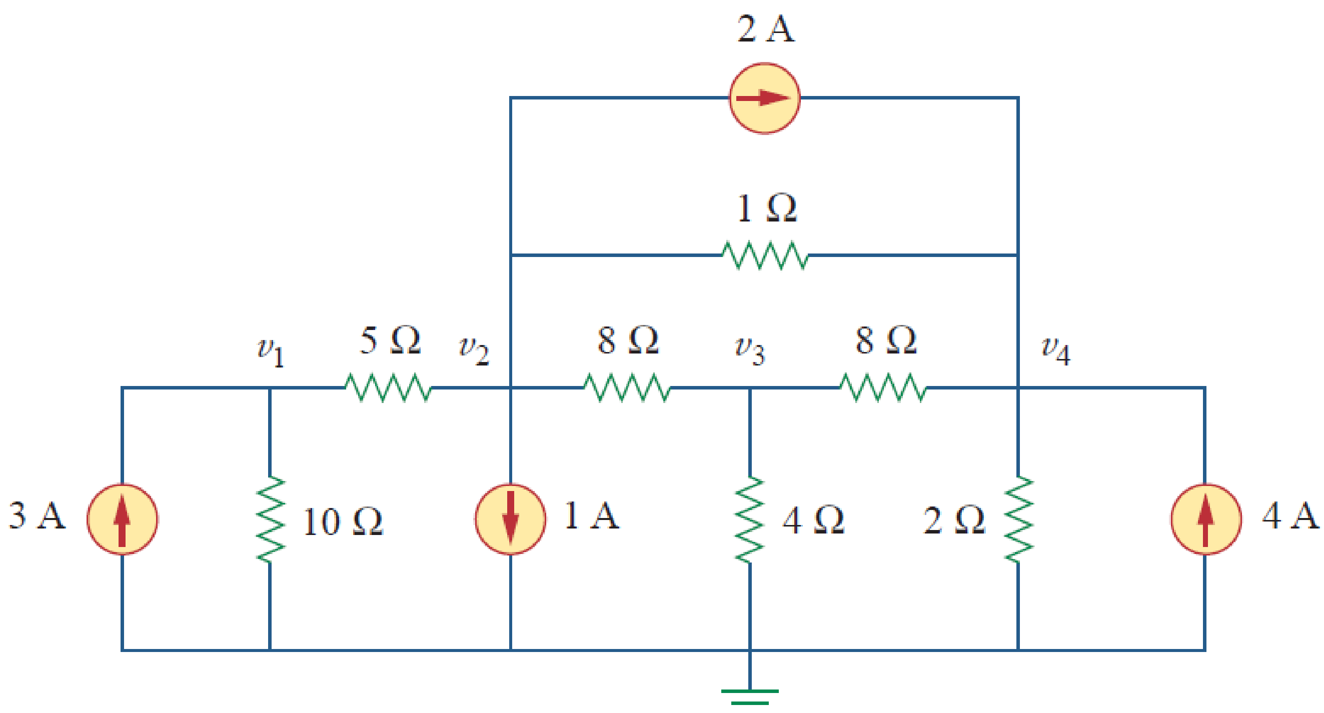

Figure 3.27
For Example 3.8.

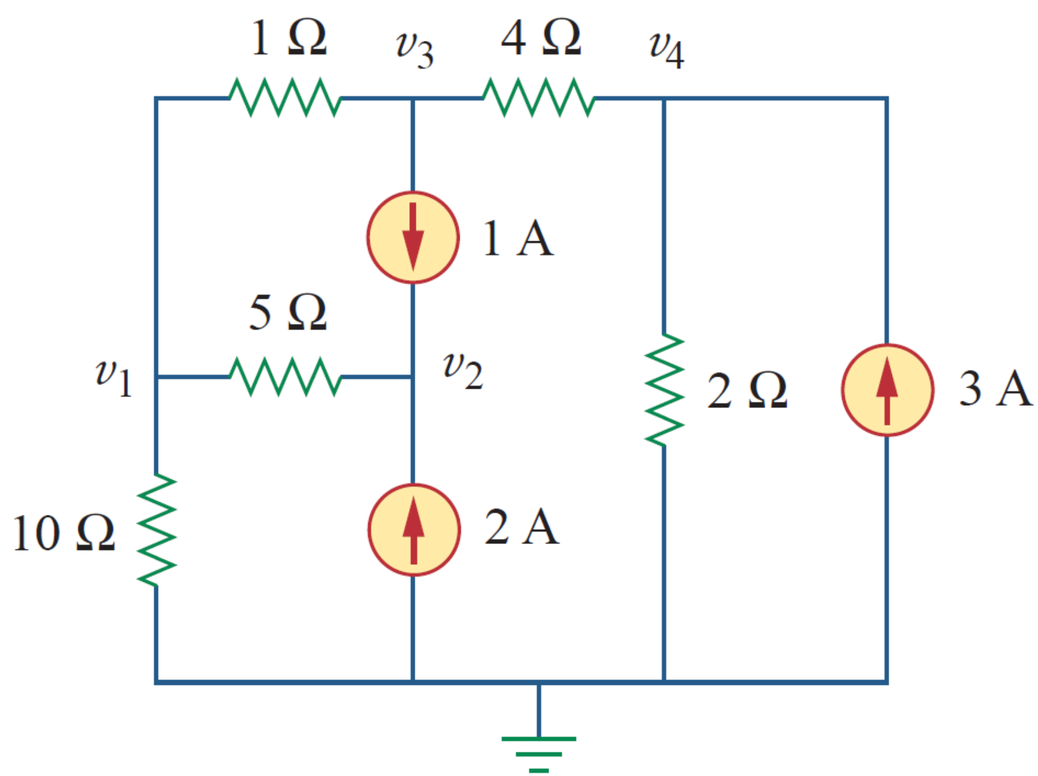

Figure 3.28
For Practice Prob. 3.8.

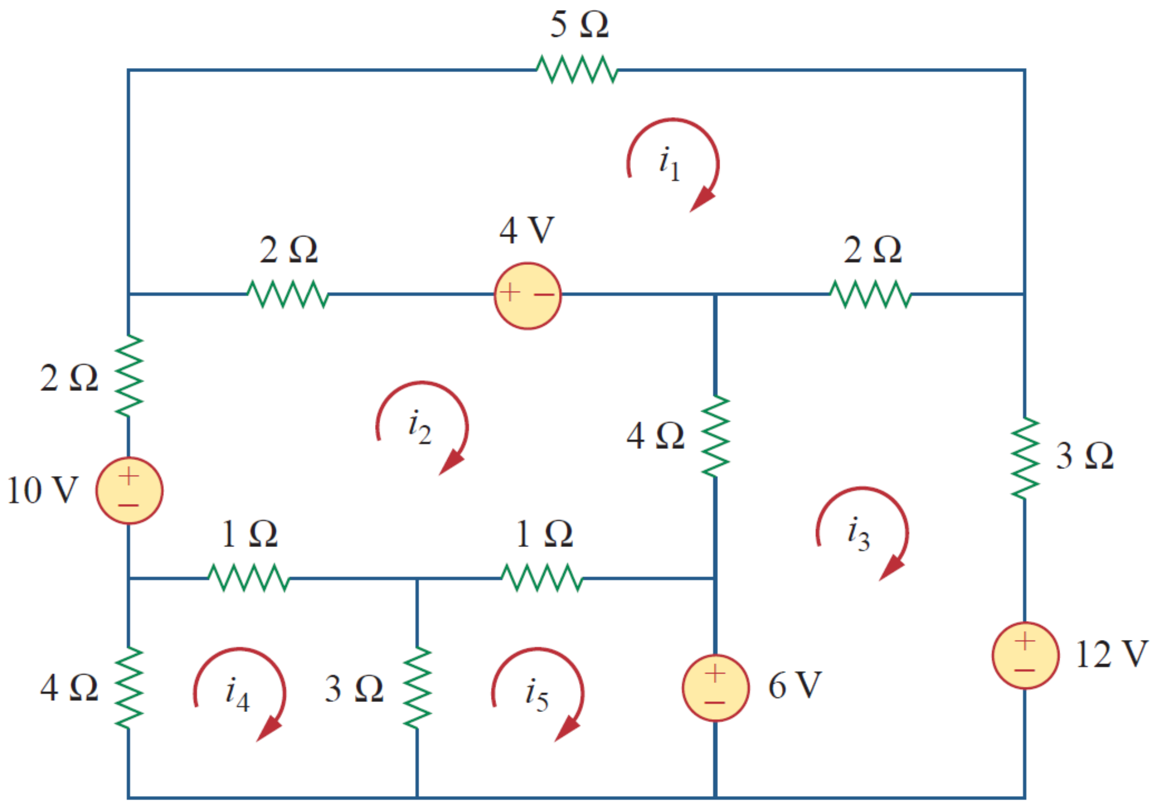

Figure 3.29
For Example 3.9.

$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

- The first factor is the nature of the particular network. Networks
- that contain many series-connected elements, voltage sources, or supermeshes are more suitable for mesh analysis, whereas networks with
- parallel-connected elements, current sources, or supernodes are more
- suitable for nodal analysis. Also, a circuit with fewer nodes than
- meshes is better analyzed using nodal analysis, while a circuit with
- fewer meshes than nodes is better analyzed using mesh analysis. The
- key is to select the method that results in the smaller number of
- equations.
- The second factor is the information required. If node voltages are
- required, it may be expedient to apply nodal analysis. If branch or mesh
- currents are required, it may be better to use mesh analysis.

Putting Eqs. (E.3.1) to (E.3.4) together in matrix form, we have

$$\begin{bmatrix} 9 & -4 & 0 & -2 \\ -4 & 15 & -4 & -6 \\ 0 & -4 & 10 & -2 \\ -2 & -6 & -2 & 20 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 6 \\ -12 \\ 12 \\ 0 \end{bmatrix}$$

or $\mathbf{AI} = \mathbf{B}$, where the vector $\mathbf{I}$ contains the unknown mesh currents. We now use *MATLAB* to determine $\mathbf{I}$ as follows:

```
>> A = [9 -4  0 -2; -4 15 -4 -6;
         0 -4 10 -2;  -2 -6 -2 20]

A =

     9  -4   0  -2
    -4  15  -4  -6
     0  -4  10  -2
    -2  -6  -2  20

>> B = [6 -12 12 0]'

B =

     6
   -12
    12
     0

>> I = inv(A)*B

I =

     0.5203
    -0.3555
     1.0682
     0.0522
```

Thus, $I_1 = 0.5203$, $I_2 = -0.3555$, $I_3 = 1.0682$, and $I_4 = 0.0522$ A.

Inspection with controlled source