Chapter 6

Capacitors and Inductors

A CAPACITOR CONSISTS OF TWO CONDUCTING PLATES SEPARATED BY AN INSULATOR (OR DIELECTRIC).

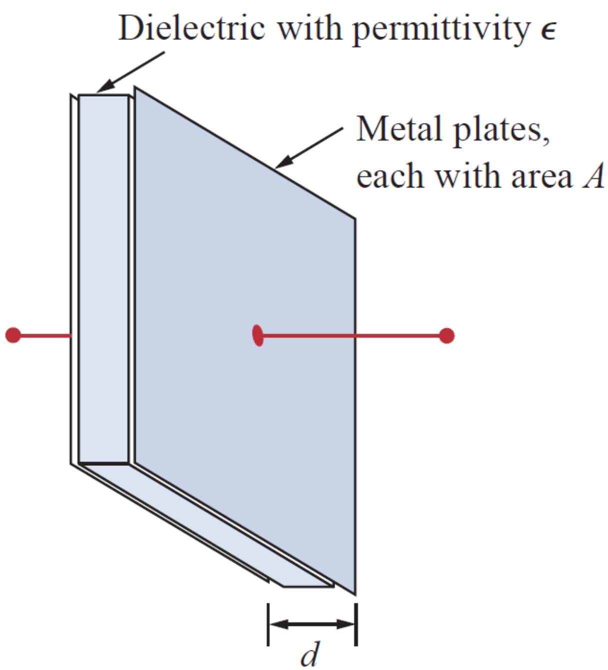


Figure 6.1
A typical capacitor.

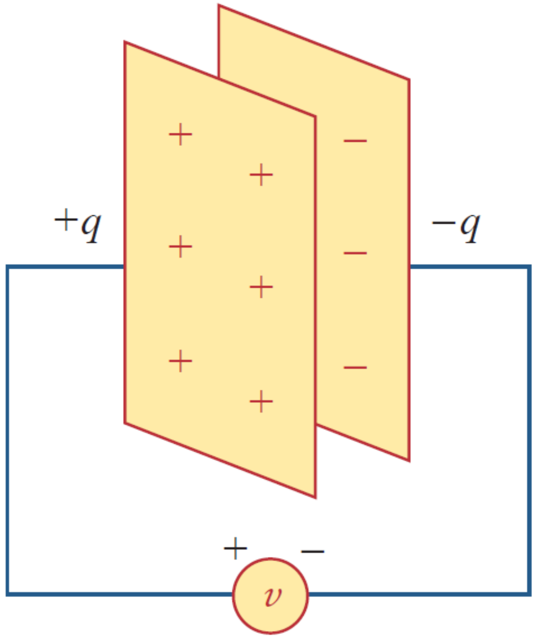


Figure 6.2
A capacitor with applied voltage v.

$$\boxed{q = Cv}$$

Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

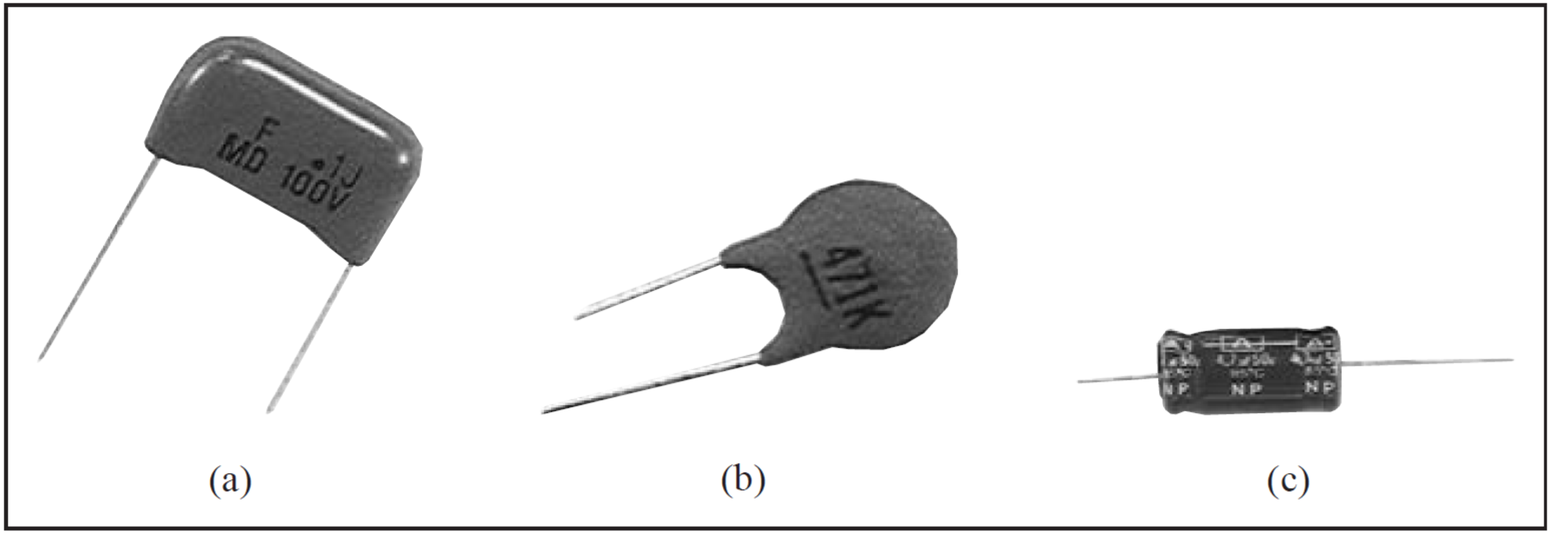


Figure 6.4

Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor.
Courtesy of Tech America.

Although the capacitance C *of a capacitor is the ratio of the charge q per plate to the applied voltage it does not depend on q or It* depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown in Fig. 6.1, the capacitance is given by

$$C = \frac{\epsilon A}{d}$$

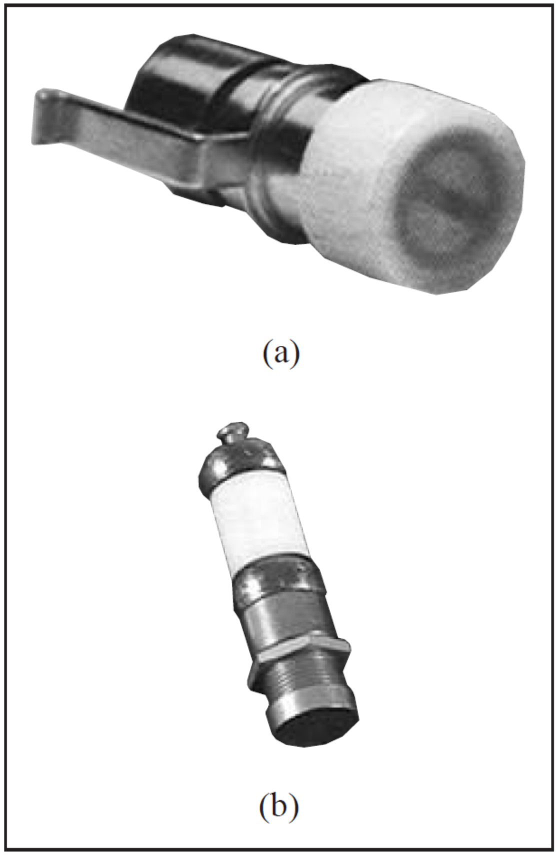


Figure 6.5
Variable capacitors: (a) trimmer capacitor, (b) filmtrim capacitor.

$$C = \frac{\epsilon A}{d}$$

where *A is the surface area of each plate, d is the distance between* the plates, and is the permittivity of the dielectric material between the plates. Although Eq. (6.2) applies to only parallel-plate capacitors, we may infer from it that, in general, three factors determine the value of the capacitance:

1. The surface area of the plates—the larger the area, the greater the capacitance.
2. The spacing between the plates—the smaller the spacing, the greater the capacitance.
3. The permittivity of the material—the higher the permittivity, the greater the capacitance.

$$i = \frac{dq}{dt}$$

$$v = \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

$$\boxed{v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)}$$

$$\boxed{i = C\frac{dv}{dt}}$$

The instantaneous power delivered to the capacitor is

$$p = vi = Cv\frac{dv}{dt} \tag{6.7}$$

The energy stored in the capacitor is therefore

$$w = \int_{-\infty}^{t} p\,dt = C\int_{-\infty}^{t} v\frac{dv}{dt}dt = C\int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)} \tag{6.8}$$

We note that $v(-\infty) = 0$, because the capacitor was uncharged at $t = -\infty$. Thus,

$$\boxed{w = \frac{1}{2}Cv^2} \tag{6.9}$$

Using Eq. (6.1), we may rewrite Eq. (6.9) as

$$w = \frac{q^2}{2C} \tag{6.10}$$

- A capacitor is an open circuit to dc.
- The voltage on a capacitor cannot change abruptly.

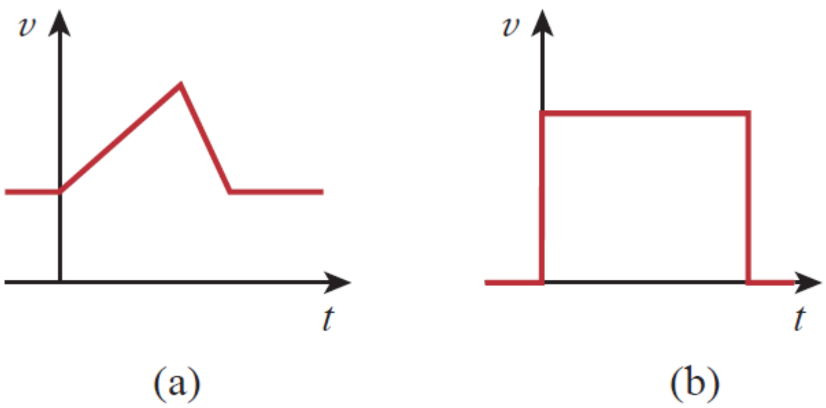


Figure 6.7
Voltage across a capacitor: (a) allowed, (b) not allowable; an abrupt change is not possible.

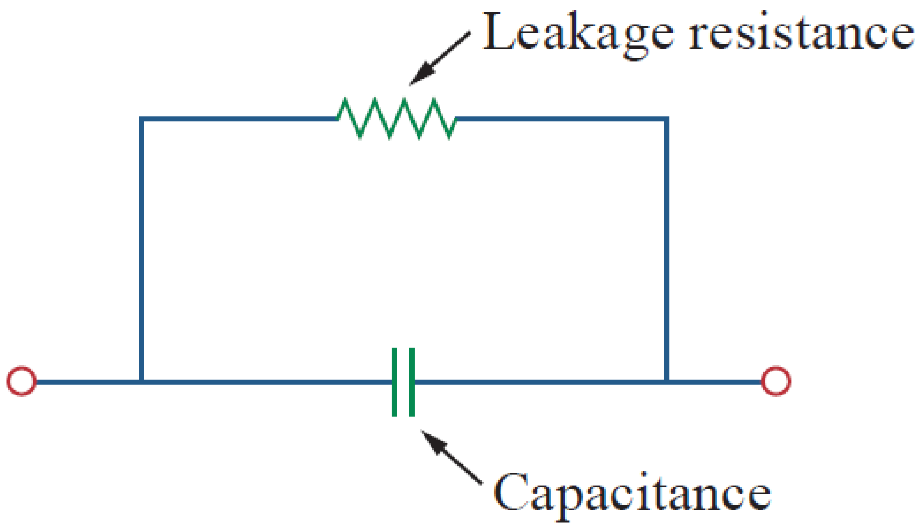
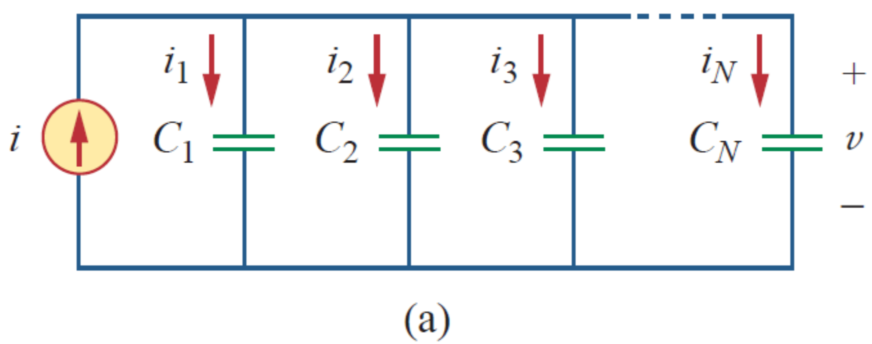


Figure 6.8
Circuit model of a nonideal capacitor.

Series and Parallel Capacitors



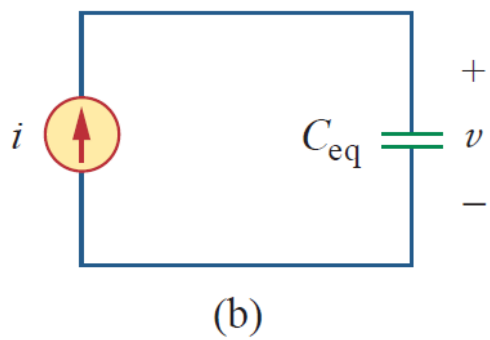
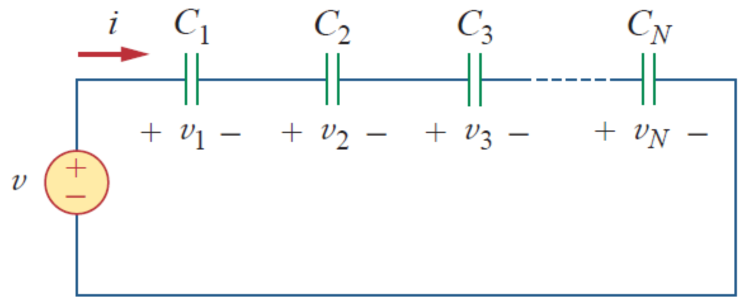
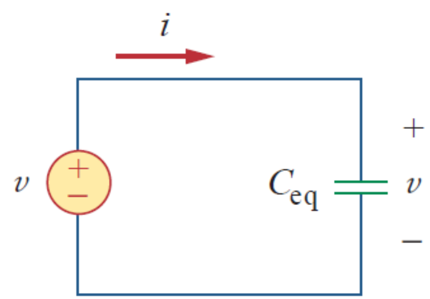


Figure 6.14
(a) Parallel-connected N capacitors,
(b) equivalent circuit for the parallel capacitors.



(a)



(b)

Figure 6.15
(a) Series-connected N capacitors,
(b) equivalent circuit for the series capacitor.

$$L = \frac{N^2 \mu A}{\ell}$$

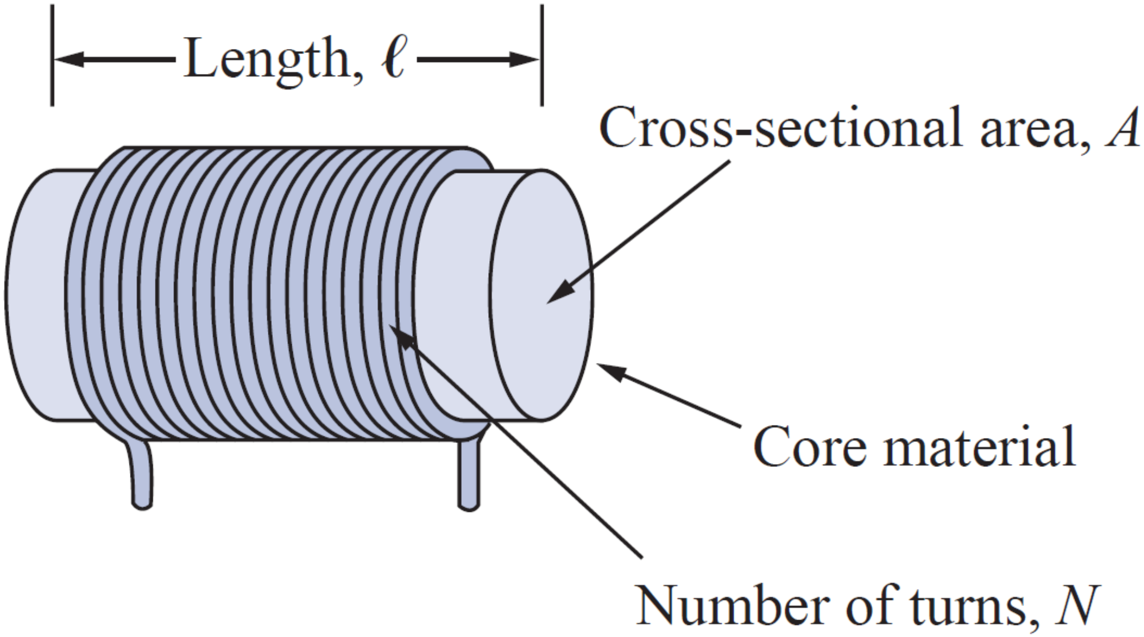


Figure 6.21
Typical form of an inductor.

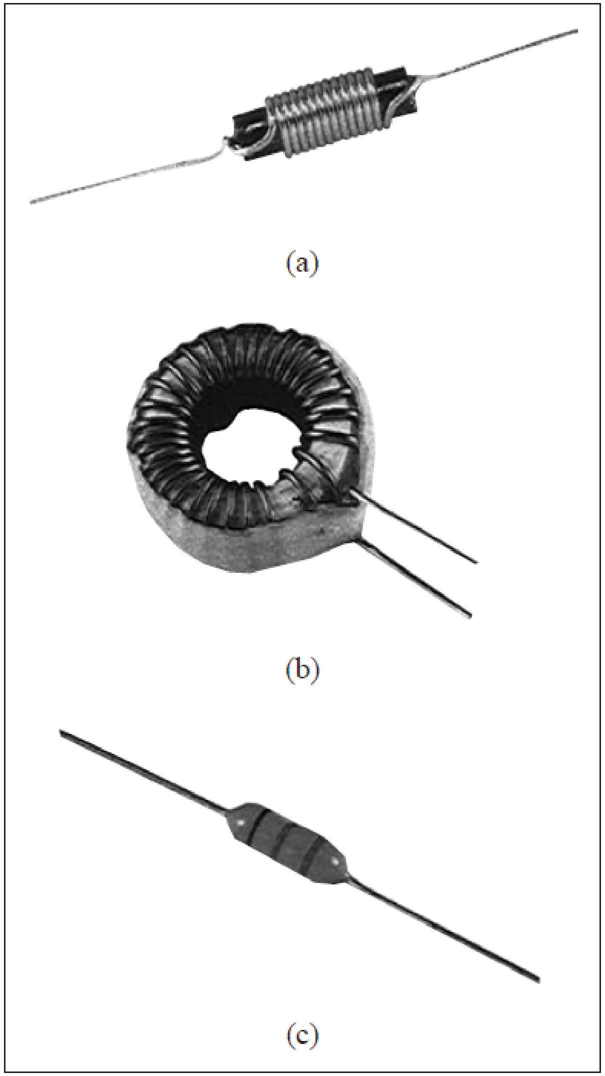


Figure 6.22
Various types of inductors: (a) solenoidal wound inductor, (b) toroidal inductor, (c) chip inductor.
Courtesy of Tech America.

$$v = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$

$$w = \int_{-\infty}^{t} p\,dt = \int_{-\infty}^{t}\left(L\frac{di}{dt}\right)i\,dt$$

$$= L\int_{-\infty}^{t} i\,di = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$

Since $i(-\infty) = 0$,

$$\boxed{w = \frac{1}{2} L i^2} \tag{6.24}$$

We should note the following important properties of an inductor.

1. Note from Eq. (6.18) that the voltage across an inductor is zero when the current is constant. Thus,

An inductor acts like a short circuit to dc.

2. An important property of the inductor is its opposition to the change in current flowing through it.

The current through an inductor cannot change instantaneously.

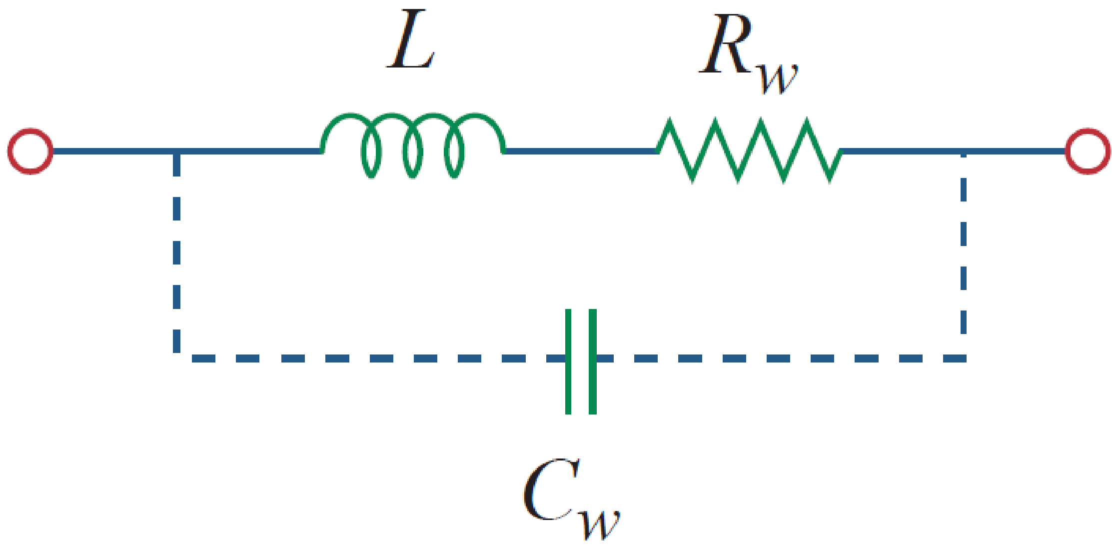


Figure 6.26
Circuit model for a practical inductor.

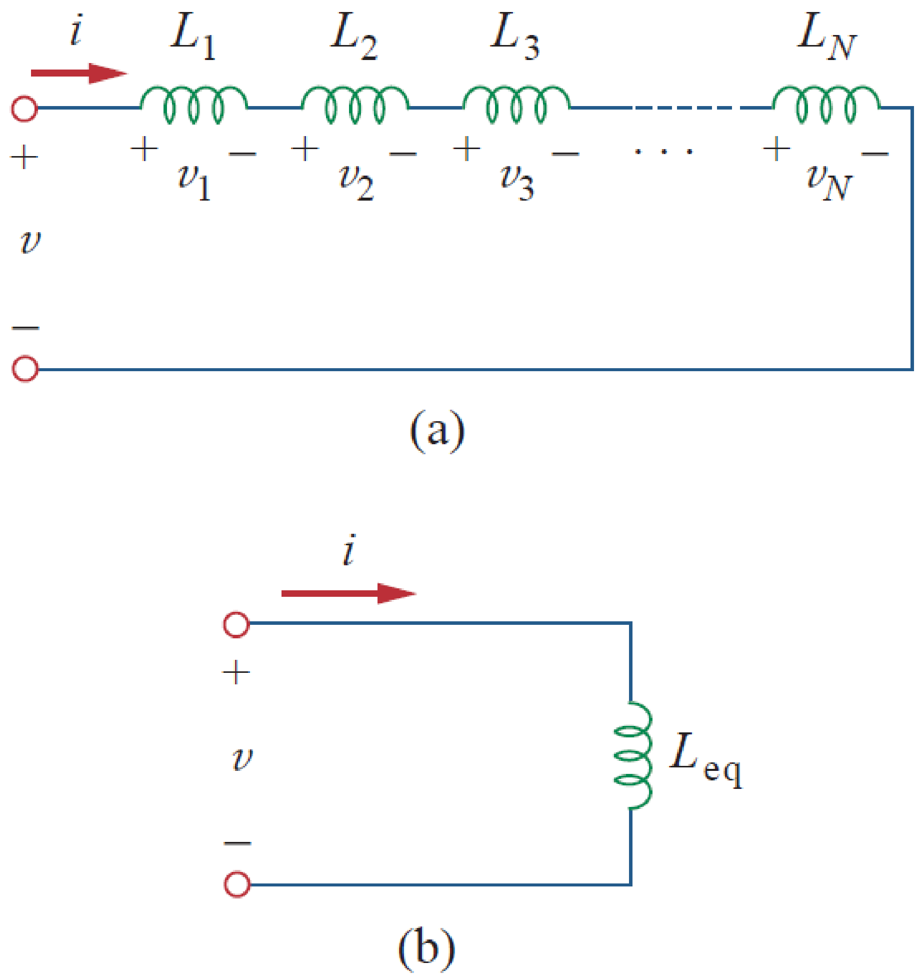
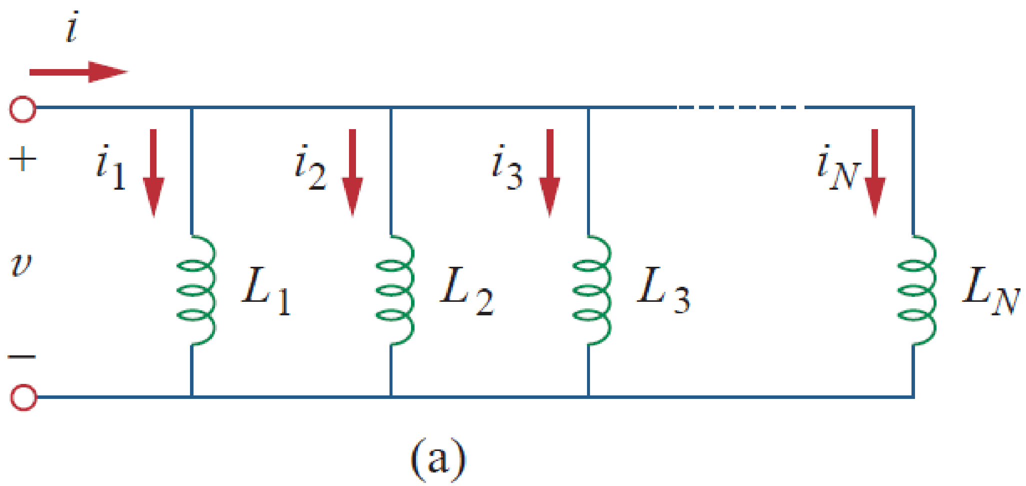


Figure 6.29

(a) A series connection of N inductors, (b) equivalent circuit for the series inductors.



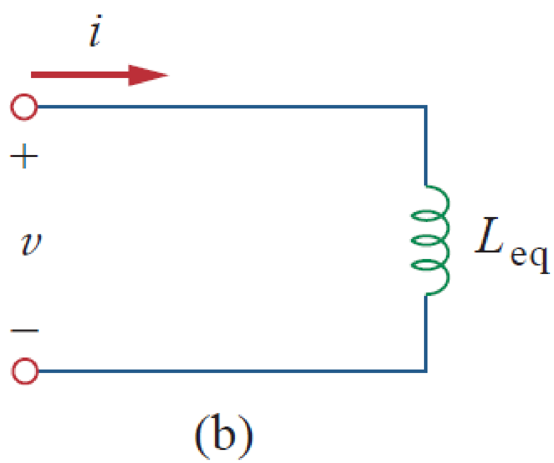


Figure 6.30

(a) A parallel connection of N inductors, (b) equivalent circuit for the parallel inductors.

TABLE 6.1

Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

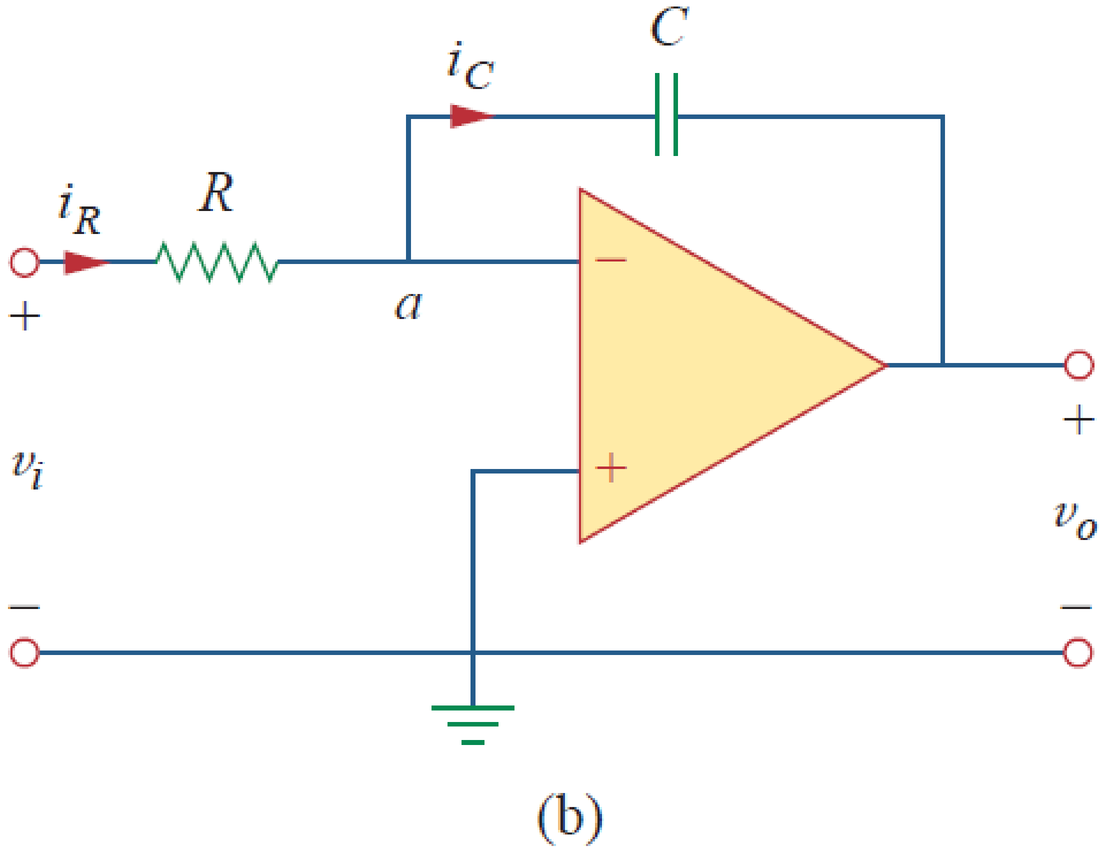


Figure 6.35
Replacing the feedback resistor in the inverting amplifier in (a) produces an integrator in (b).

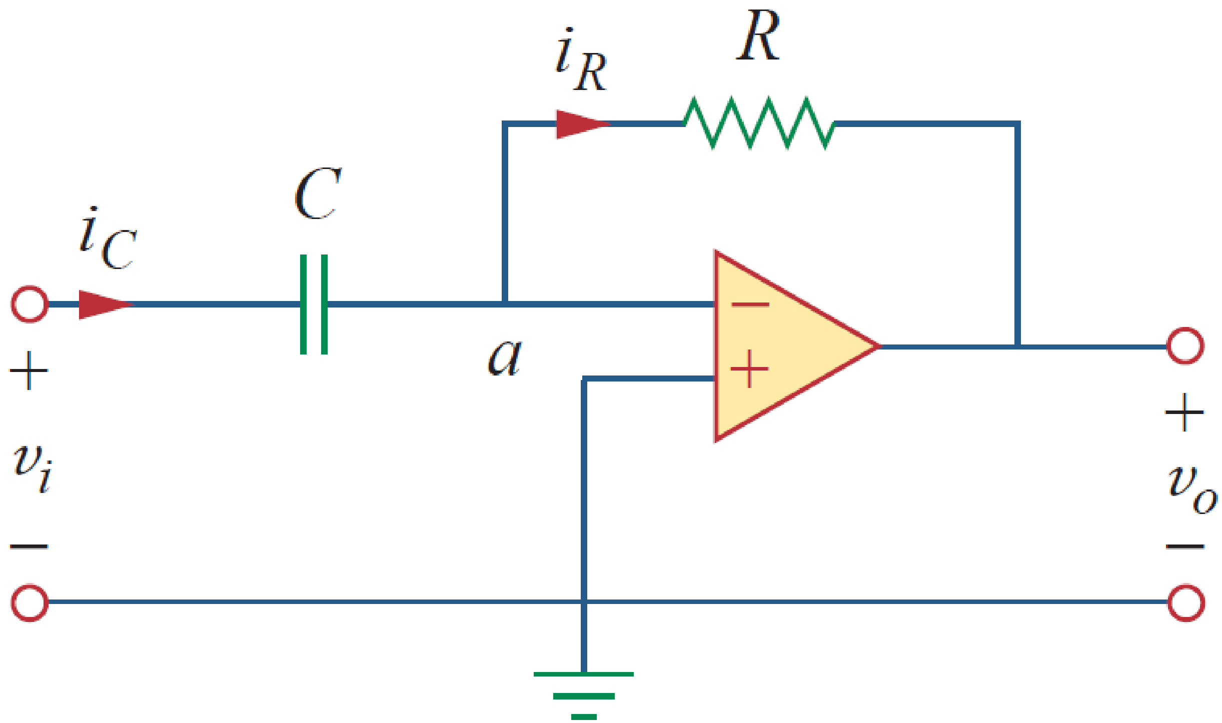


Figure 6.37
An op amp differentiator.