Chapter 13

Three-Phase Circuits

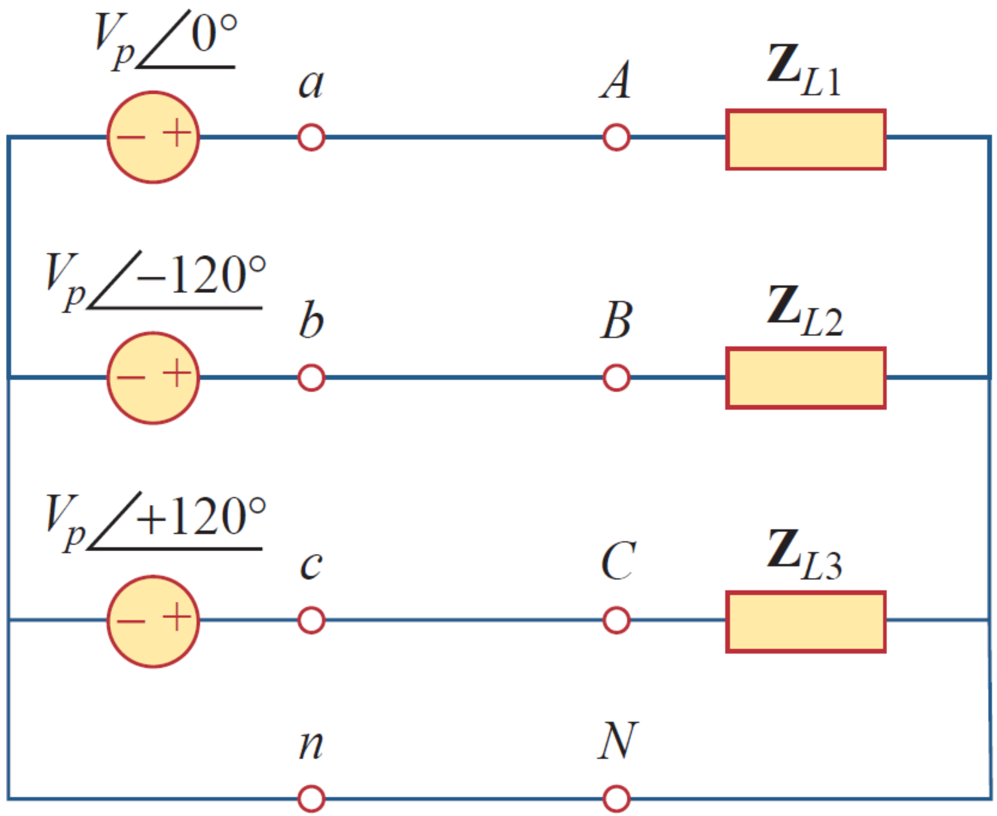


Figure 12.3
Three-phase four-wire system.

- Three-phase systems are important for at least three reasons.
- First,nearly all electric power is generated and distributed in three-phase.
- Second, the instantaneous power in a three-phase system can be constant (not pulsating), This results in uniform power transmission and less vibration of three-phase machines.
- Third, for the same amount of power, the three-phase system is more economical than the single phase.The amount of wire required for a three-phase system is less than that required for an equivalent single-phase system.

12.2 Balanced Three-Phase Voltages

Three-phase voltages are often produced with a three-phase ac generator (or alternator) whose cross-sectional view is shown in Fig. 12.4. The generator basically consists of a rotating magnet (called the *rotor*) surrounded by a stationary winding (called the *stator*). Three separate

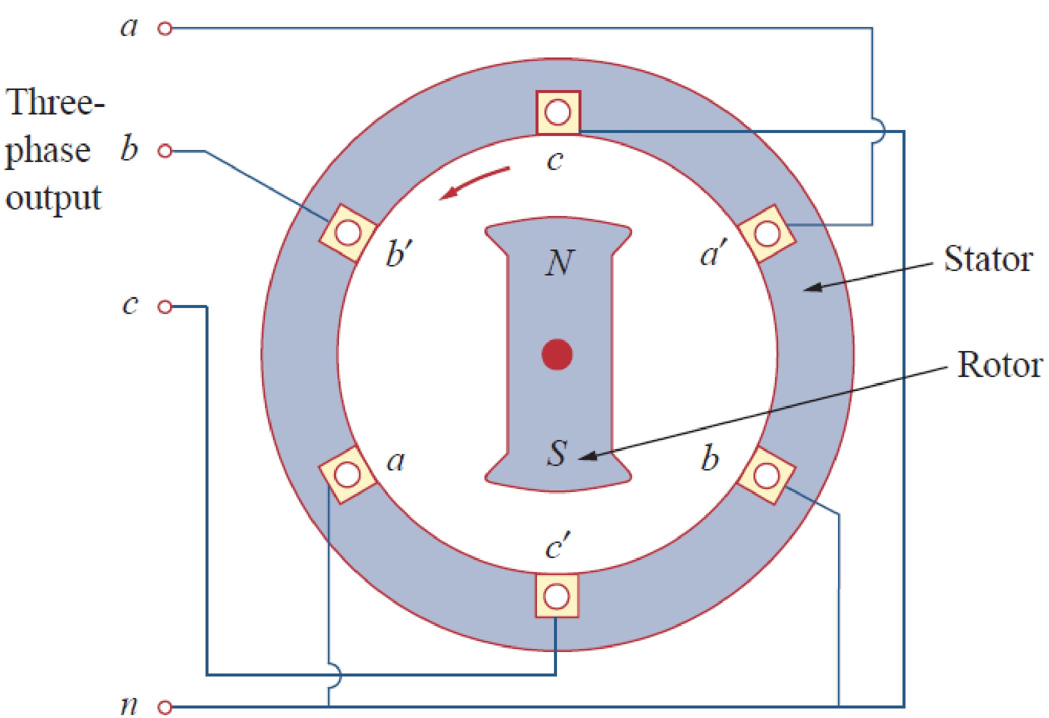


Figure 12.4
A three-phase generator.

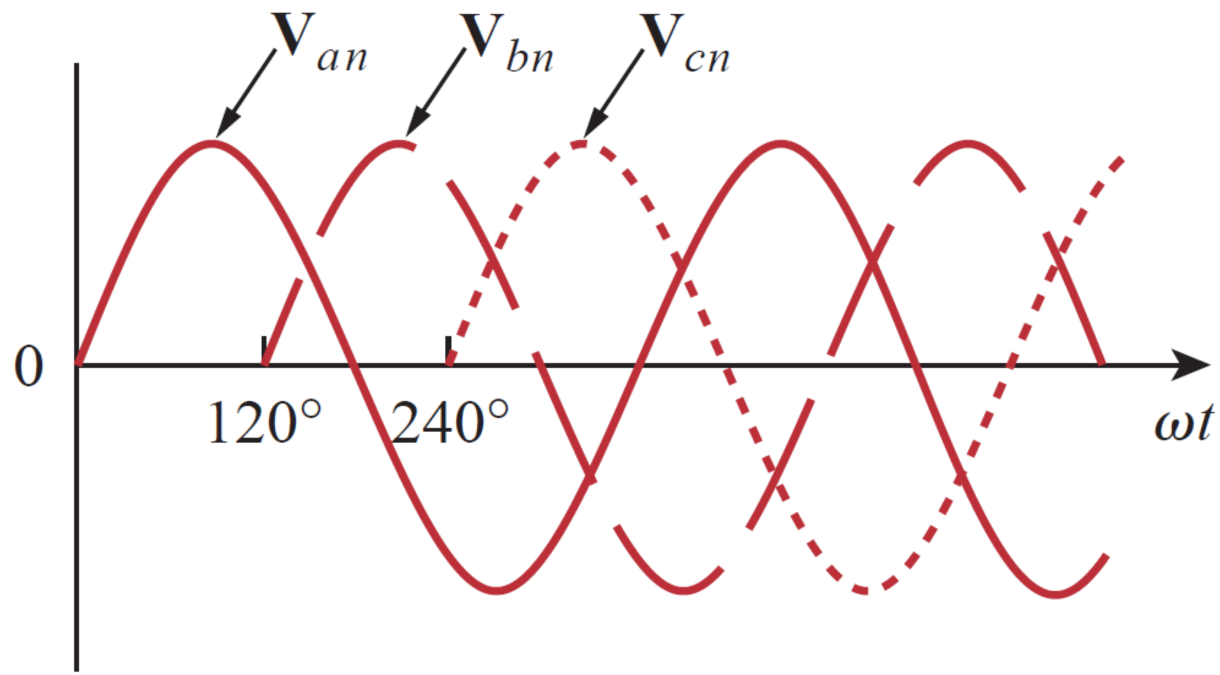


Figure 12.5

The generated voltages are 120° apart from each other.

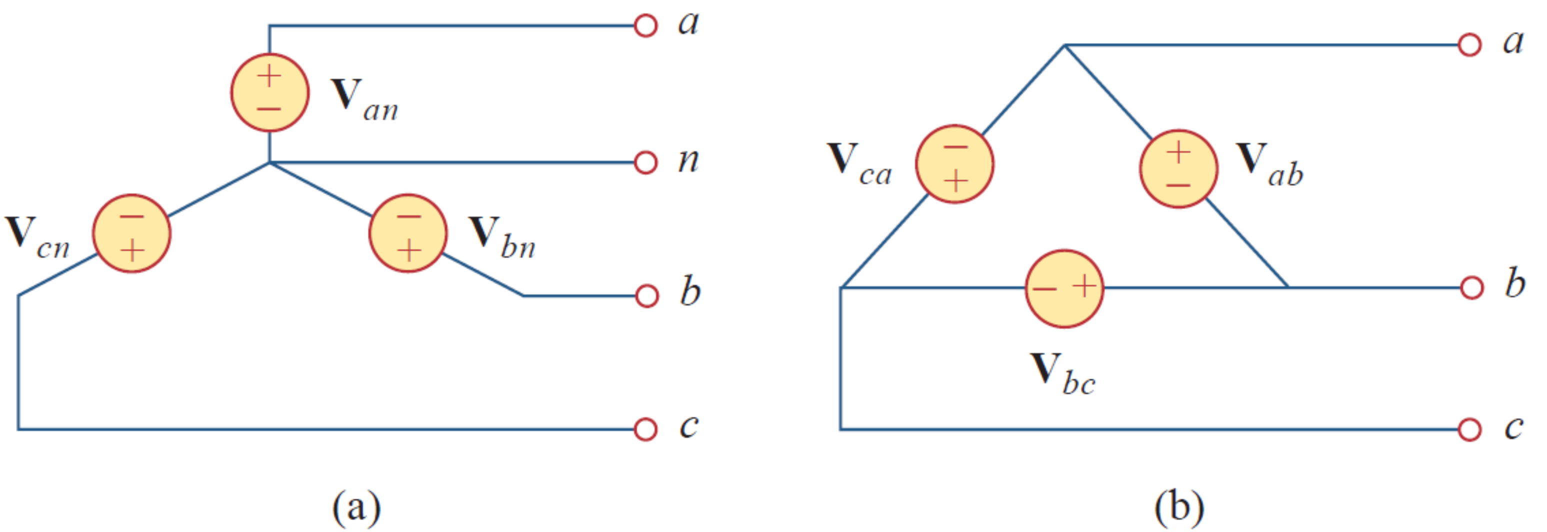


Figure 12.6

Three-phase voltage sources: (a) Y-connected source, (b) Δ-connected source.

Balanced phase voltages are equal in magnitude and are out of phase with each other by 120°.

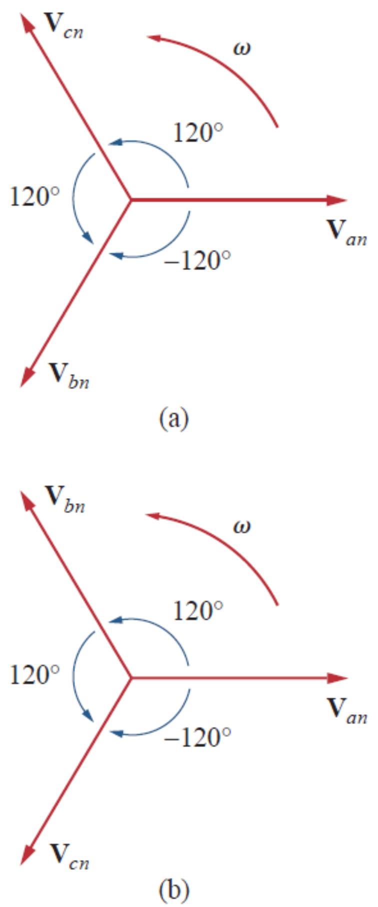


Figure 12.7
Phase sequences: (a) *abc* or positive sequence, (b) *acb* or negative sequence.

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

where V_p is the effective or rms value of the phase voltages. This is known as the *abc sequence* or *positive sequence*. In this phase sequence, $\mathbf{V}_{an}$ leads $\mathbf{V}_{bn}$, which in turn leads $\mathbf{V}_{cn}$. This sequence is produced when the rotor in Fig. 12.4 rotates counterclockwise. The other possibility is shown in Fig. 12.7(b) and is given by

$$\boxed{\begin{aligned} \mathbf{V}_{an} &= V_p\angle 0^\circ \\ \mathbf{V}_{cn} &= V_p\angle -120^\circ \\ \mathbf{V}_{bn} &= V_p\angle -240^\circ = V_p\angle +120^\circ \end{aligned}} \tag{12.4}$$

$$\begin{aligned} \mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} &= V_p\angle 0^\circ + V_p\angle -120^\circ + V_p\angle +120^\circ \\ &= V_p(1.0 - 0.5 - j0.866 - 0.5 + j0.866) \\ &= 0 \end{aligned}$$

A **balanced load** is one in which the phase impedances are equal in magnitude and in phase.

Reminder: A Y-connected load consists of three impedances connected to a neutral node, while a Δ-connected load consists of three impedances connected around a loop. The load is balanced when the three impedances are equal in either case.

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y$$

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta$$

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad \text{or} \qquad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta$$

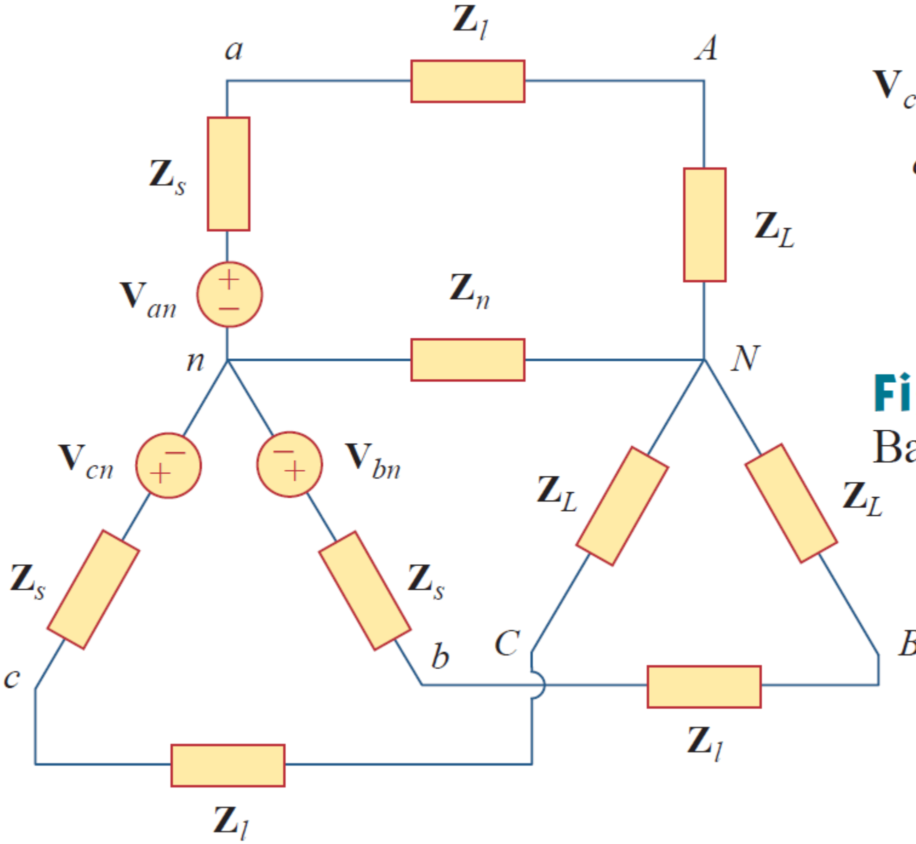


Figure 12.9
A balanced Y-Y system, showing the source, line, and load impedances.

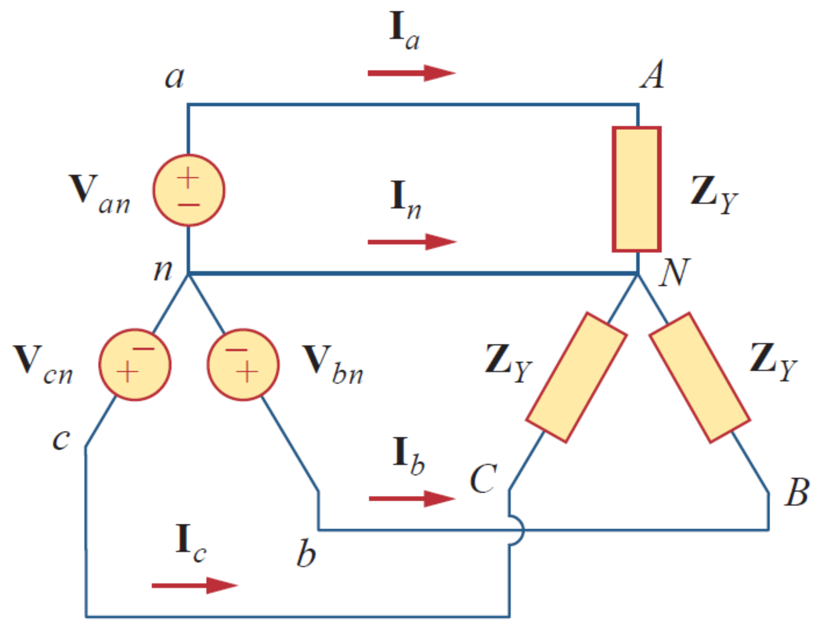


Figure 12.10
Balanced Y-Y connection.

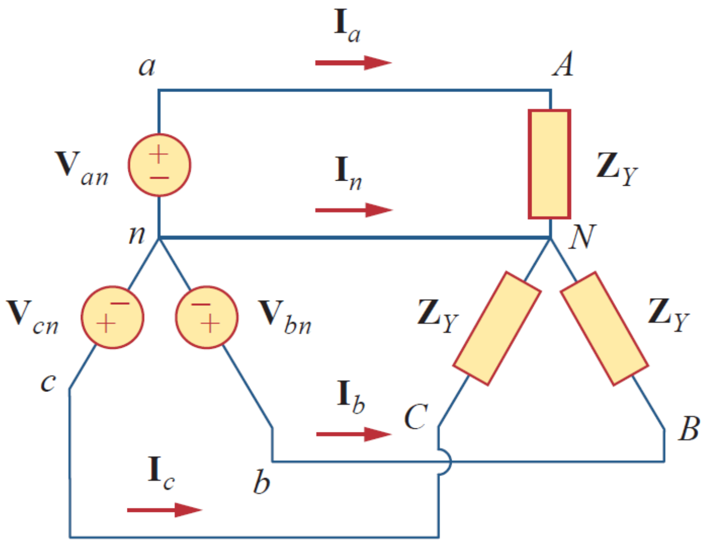
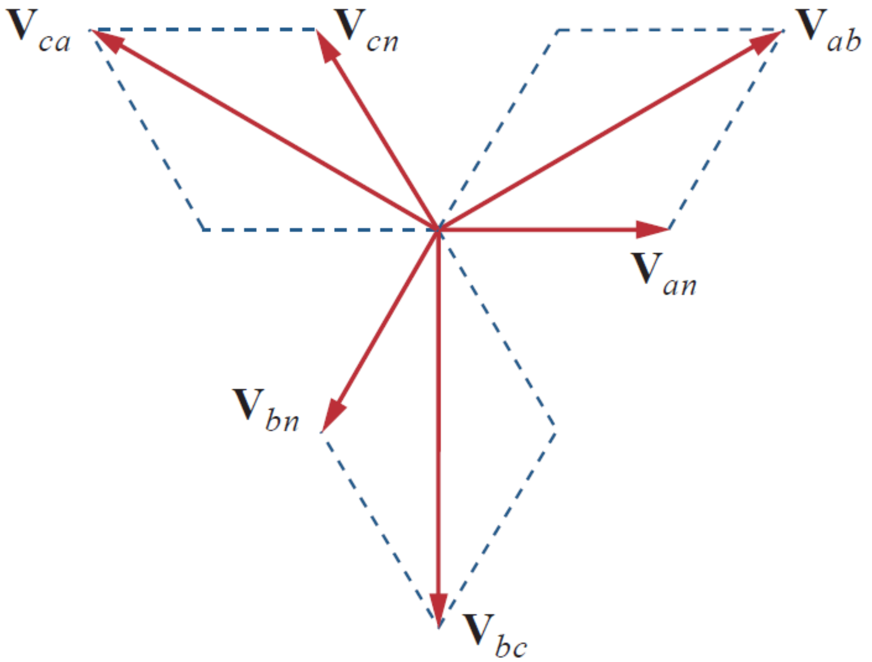


Figure 12.10
Balanced Y-Y connection.



(b)

Figure 12.11
Phasor diagrams illustrating the relationship between line voltages and phase voltages.

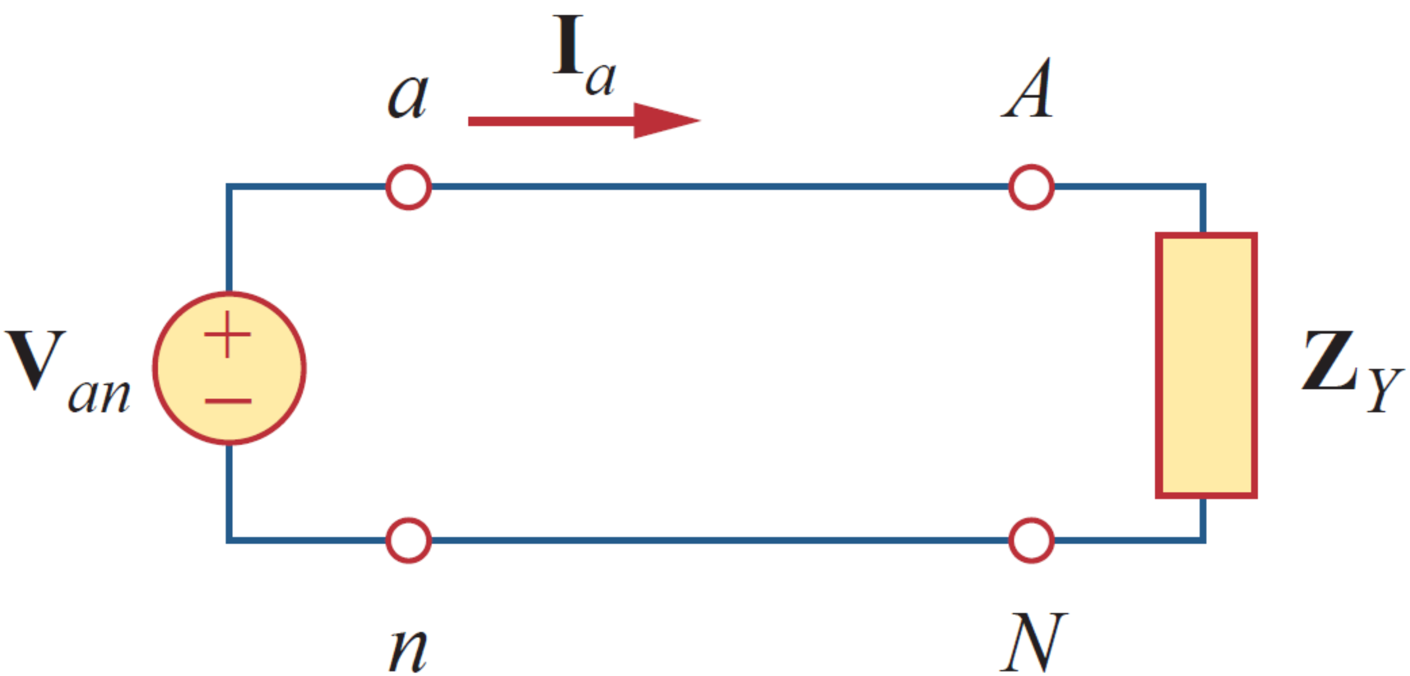


Figure 12.12
A single-phase equivalent circuit.

Calculate the line currents in the three-wire Y-Y system of Fig. 12.13.

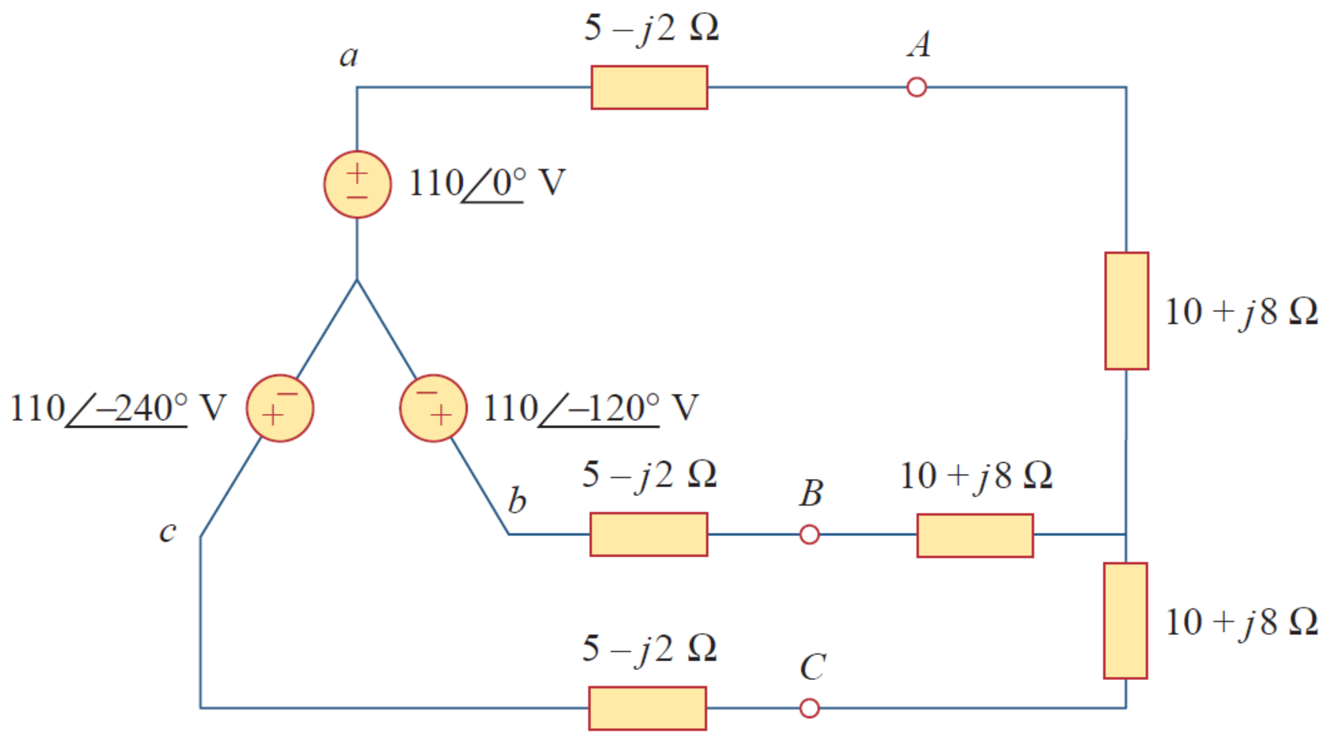


Figure 12.13
Three-wire Y-Y system; for Example 12.2.

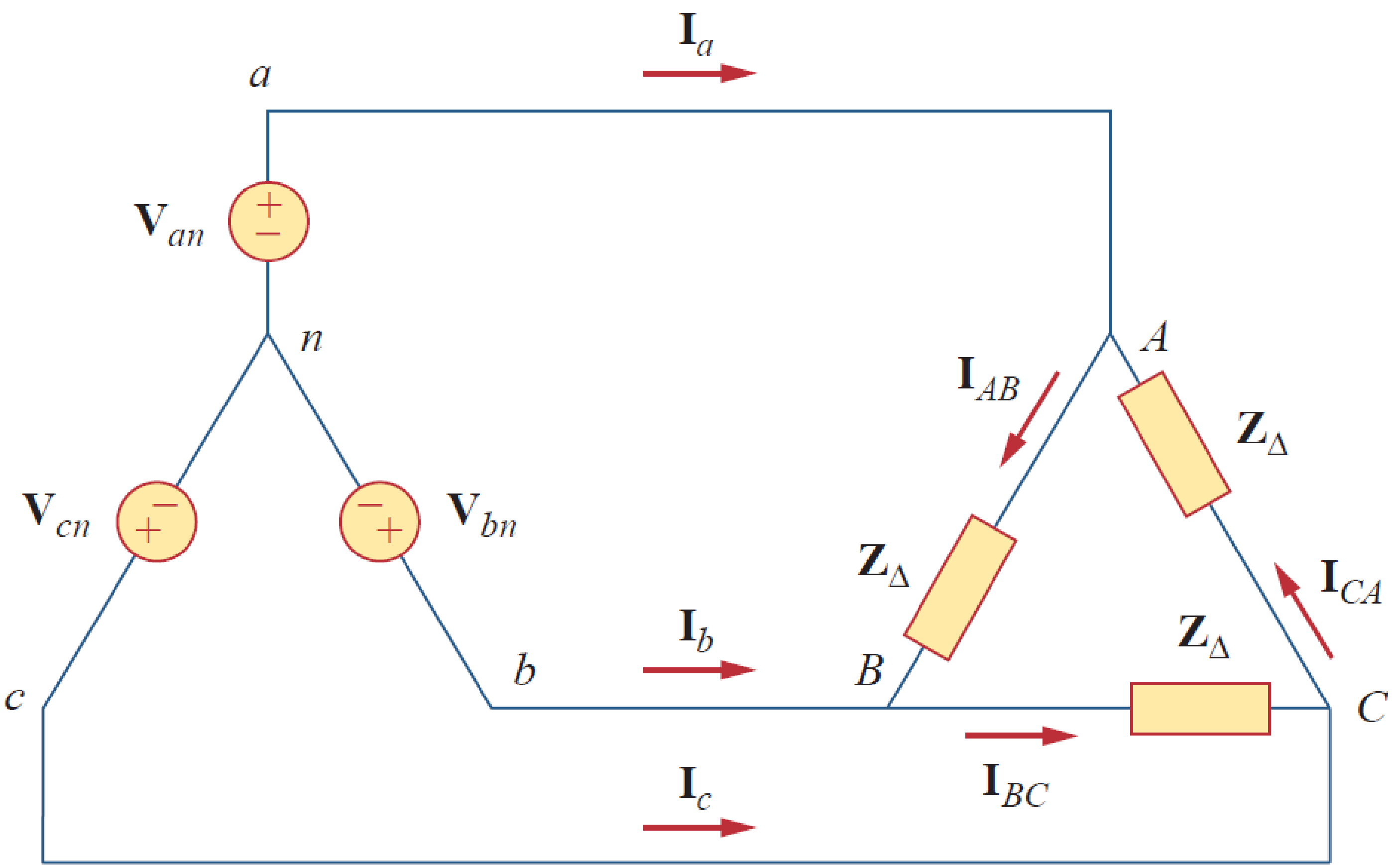


Figure 12.14
Balanced Y-Δ connection.

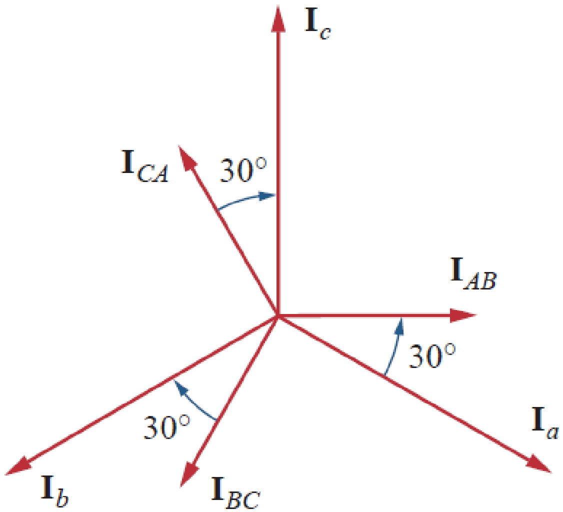


Figure 12.15
Phasor diagram illustrating the relationship between phase and line currents.

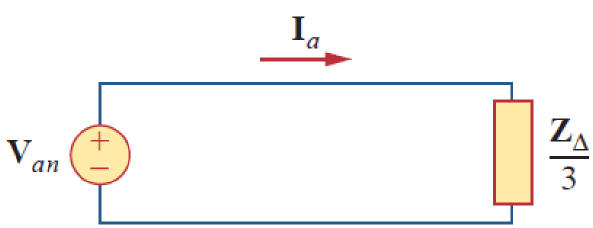


Figure 12.16
A single-phase equivalent circuit of a balanced Y-Δ circuit.

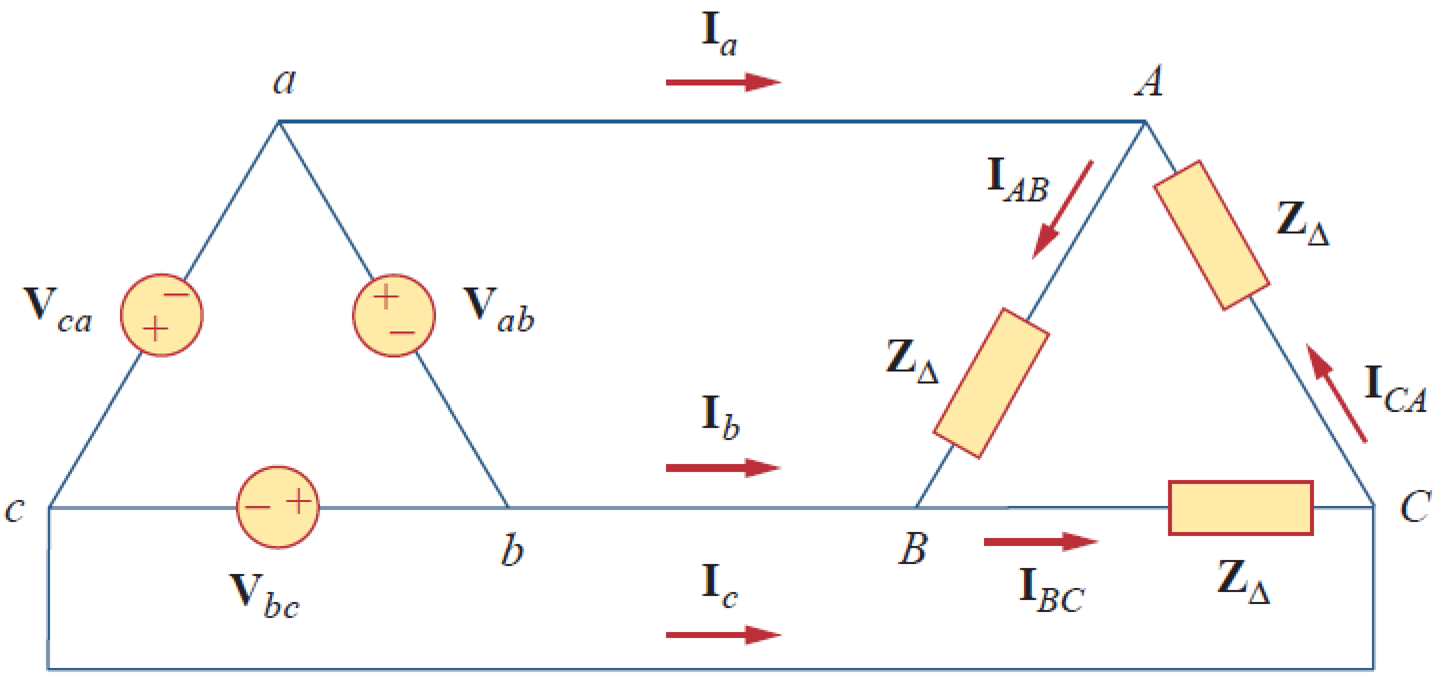


Figure 12.17
A balanced Δ-Δ connection.

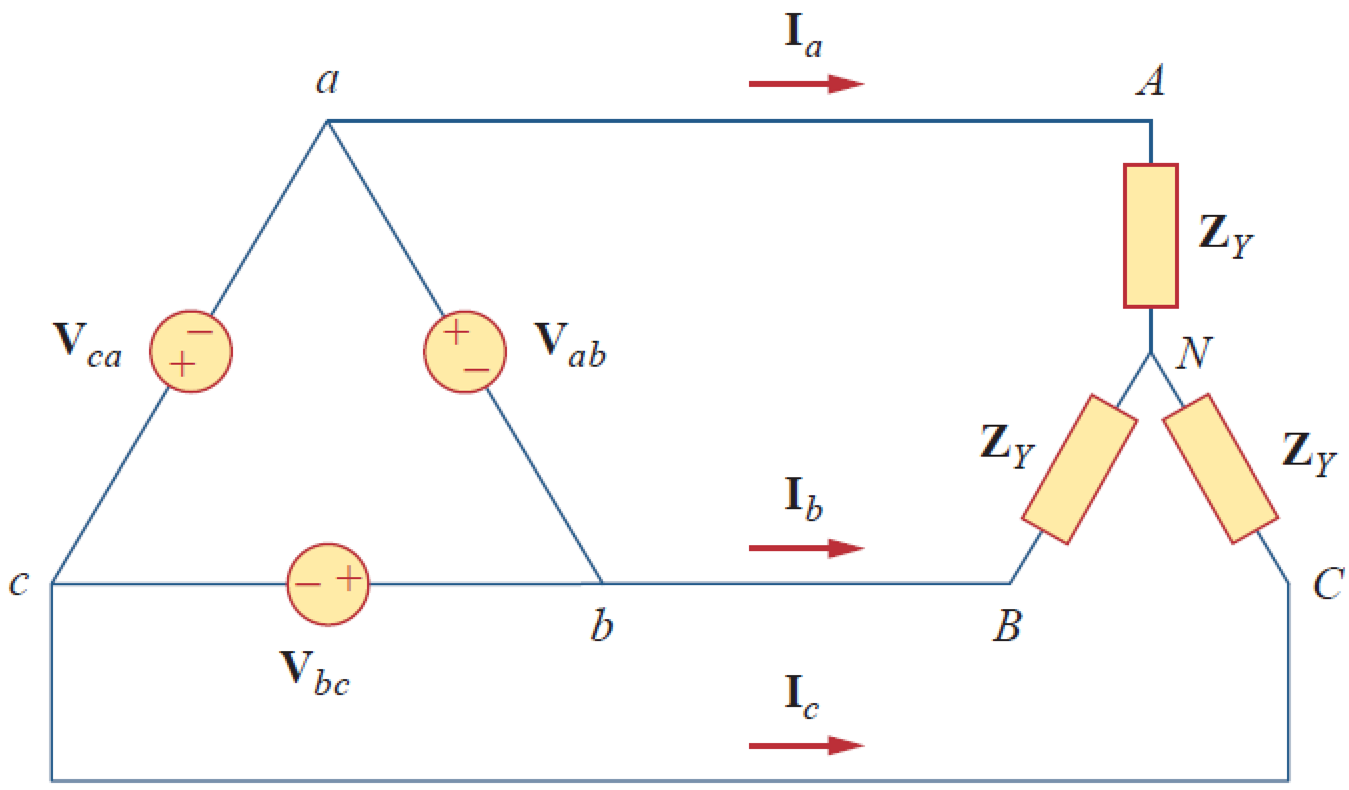


Figure 12.18

A balanced Δ-Y connection.

12.15 The circuit in Fig. 12.48 is excited by a balanced three-phase source with a line voltage of 210 V. If $\mathbf{Z}_l = 1 + j1\ \Omega$, $\mathbf{Z}_\Delta = 24 - j30\ \Omega$, and $\mathbf{Z}_Y = 12 + j5\ \Omega$, determine the magnitude of the line current of the combined loads.

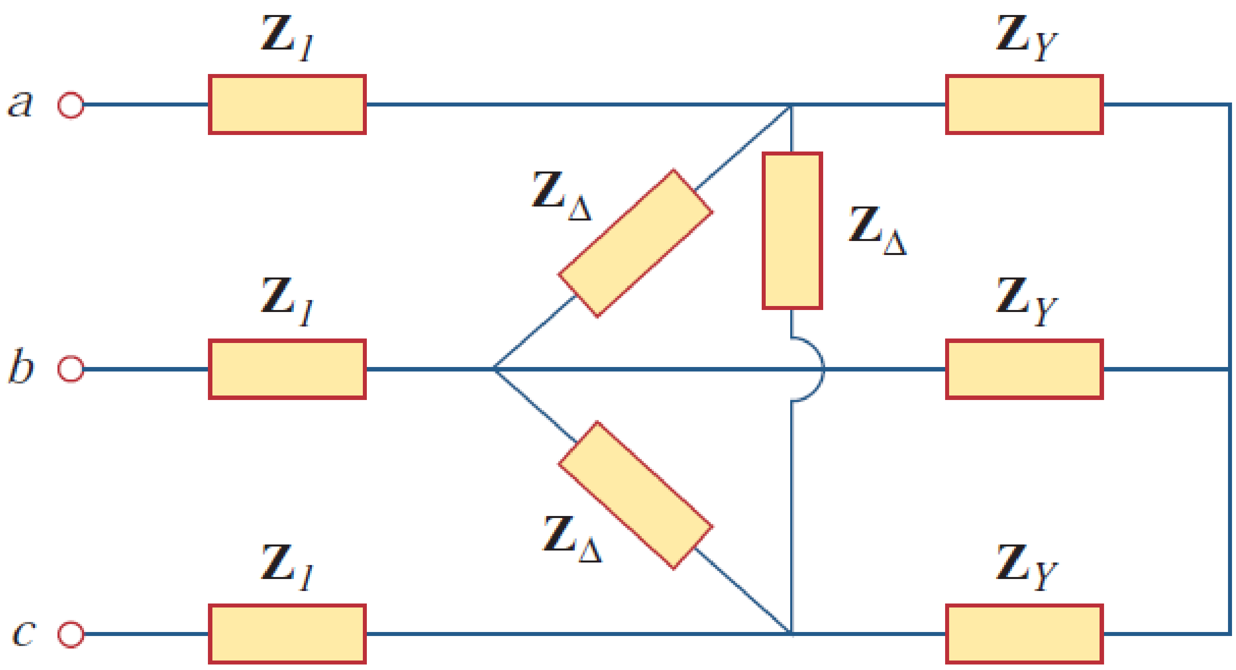


Figure 12.48
For Prob. 12.15.

12.22 Find the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ in the three-phase network of Fig. 12.53 below. Take $\mathbf{Z}_\Delta = 12 - j15\ \Omega$, $\mathbf{Z}_Y = 4 + j6\ \Omega$, and $\mathbf{Z}_l = 2\ \Omega$.

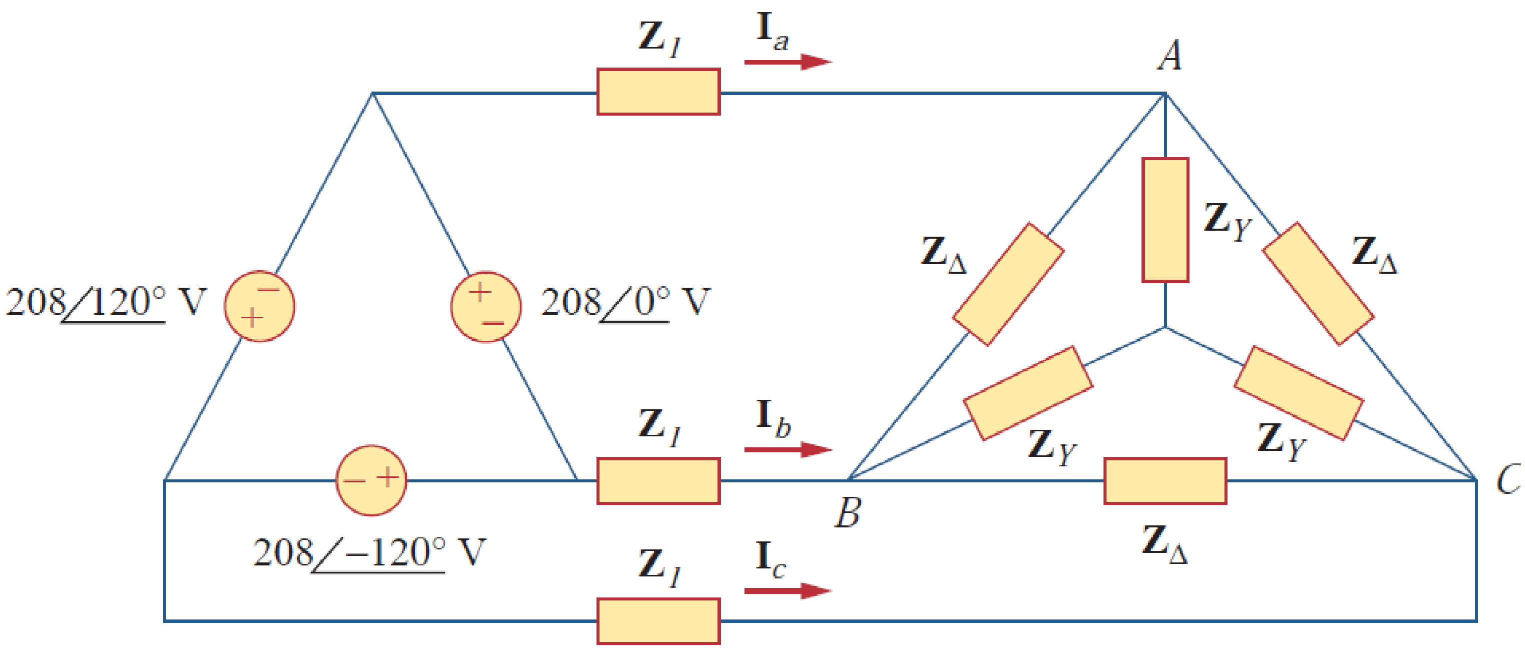


Figure 12.53
For Prob. 12.22.

POWER IN A BALANCED SYSTEM

$$v_{AN} = \sqrt{2}V_p \cos\omega t, \qquad v_{BN} = \sqrt{2}V_p \cos(\omega t - 120°)$$

$$v_{CN} = \sqrt{2}V_p \cos(\omega t + 120°)$$

$$i_a = \sqrt{2}I_p \cos(\omega t - \theta), \qquad i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120°)$$

$$i_c = \sqrt{2}I_p \cos(\omega t - \theta + 120°)$$

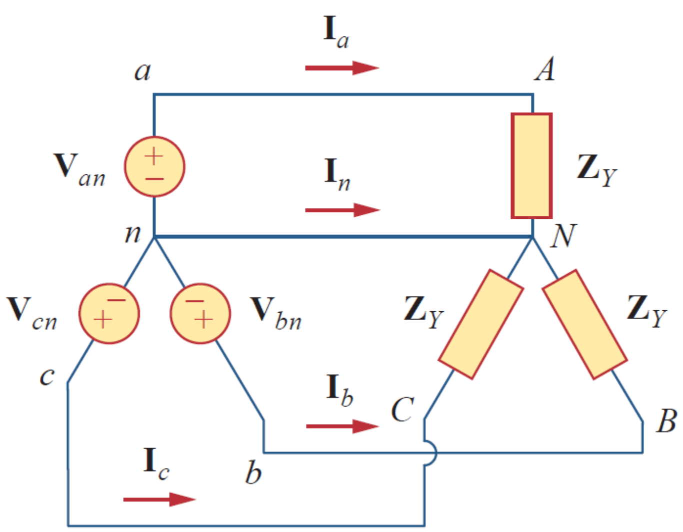


Figure 12.10
Balanced Y-Y connection.

For balanced system, current in neutral line is zero. The neutral line existing or not will have the same result.

$$
\begin{aligned}
p = p_a + p_b + p_c &= v_{AN} i_a + v_{BN} i_b + v_{CN} i_c \\
&= 2V_p I_p [\cos \omega t \cos(\omega t - \theta) \\
&\qquad + \cos(\omega t - 120^\circ) \cos(\omega t - \theta - 120^\circ) \\
&\qquad + \cos(\omega t + 120^\circ) \cos(\omega t - \theta + 120^\circ)]
\end{aligned}
\tag{12.43}
$$

Applying the trigonometric identity

$$
\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)] \tag{12.44}
$$

gives

$$
\begin{aligned}
p &= V_p I_p [3 \cos \theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240^\circ) \\
&\qquad + \cos(2\omega t - \theta + 240^\circ)] \\
&= V_p I_p [3 \cos \theta + \cos \alpha + \cos \alpha \cos 240^\circ + \sin \alpha \sin 240^\circ \\
&\qquad + \cos \alpha \cos 240^\circ - \sin \alpha \sin 240^\circ] \\
&\qquad \text{where } \alpha = 2\omega t - \theta \\
&= V_p I_p \left[3 \cos \theta + \cos \alpha + 2\left(-\frac{1}{2}\right)\cos \alpha \right] = 3 V_p I_p \cos \theta
\end{aligned}
\tag{12.45}
$$

$$P_p = V_p I_p \cos\theta \tag{12.46}$$

and the reactive power per phase is

$$Q_p = V_p I_p \sin\theta \tag{12.47}$$

The apparent power per phase is

$$S_p = V_p I_p \tag{12.48}$$

The complex power per phase is

$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^* \tag{12.49}$$

where $\mathbf{V}_p$ and $\mathbf{I}_p$ are the phase voltage and phase current with magnitudes V_p and I_p, respectively. The total average power is the sum of the average powers in the phases:

$$P = P_a + P_b + P_c = 3P_p = 3V_p I_p \cos\theta = \sqrt{3} V_L I_L \cos\theta \tag{12.50}$$

Unbalanced Three-Phase Systems

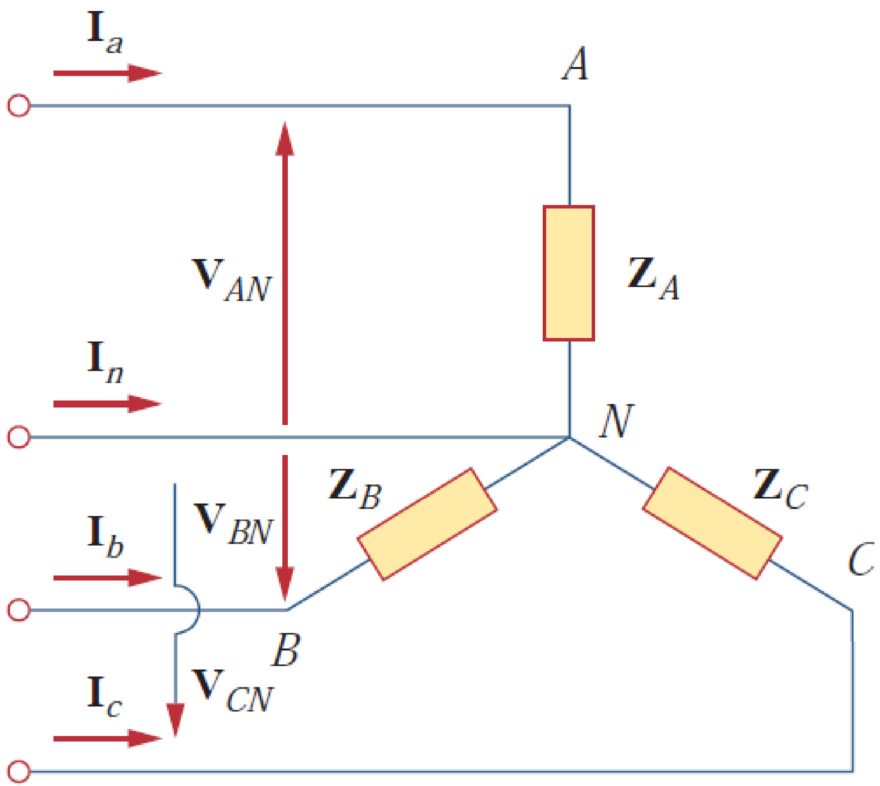


Figure 12.23
Unbalanced three-phase Y-connected load.

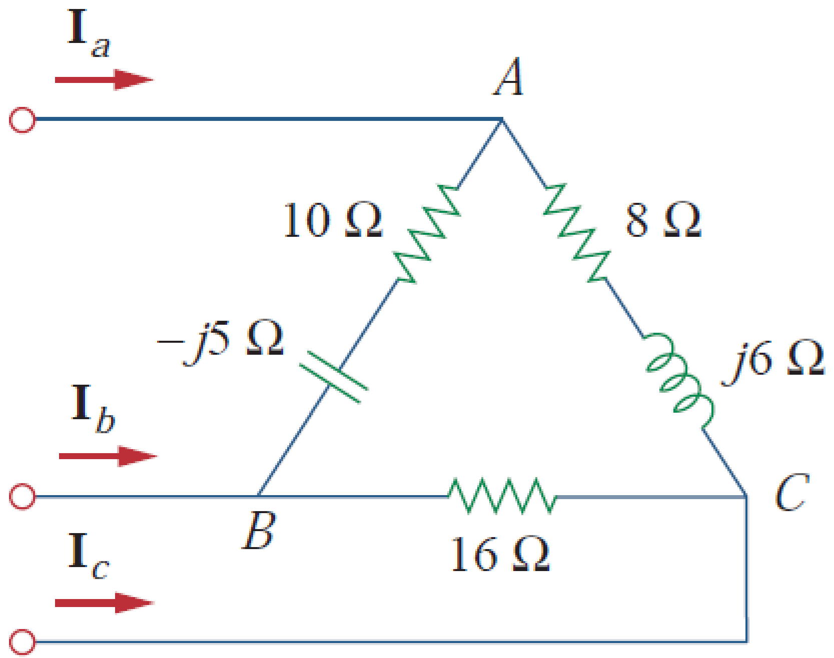


Figure 12.24

Unbalanced Δ-load; for Practice Prob. 12.9.

For the unbalanced circuit in Fig. 12.25, find: (a) the line currents, (b) the total complex power absorbed by the load, and (c) the total complex power absorbed by the source.

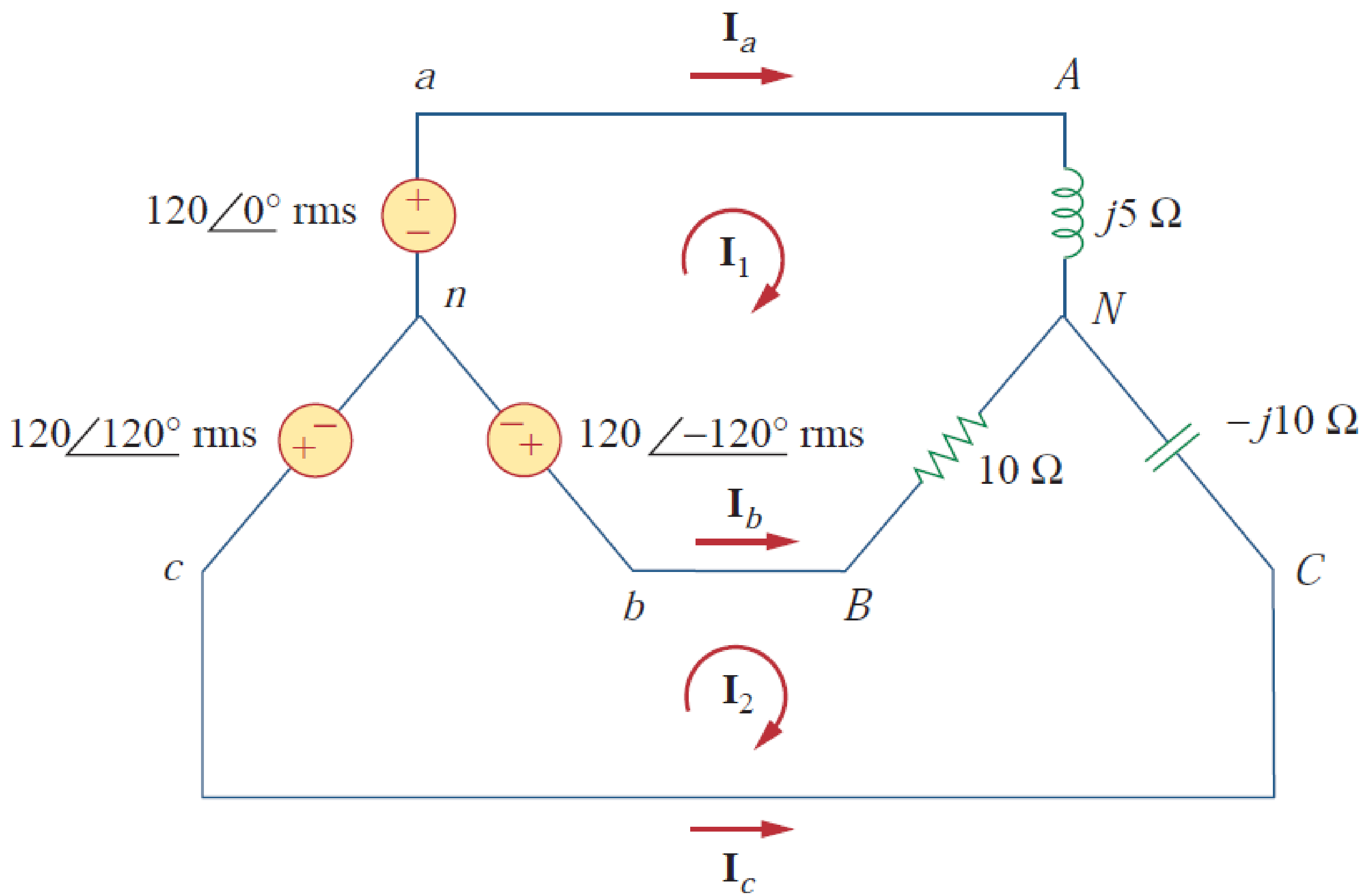


Figure 12.25
For Example 12.10.

POWER MEASURMENT

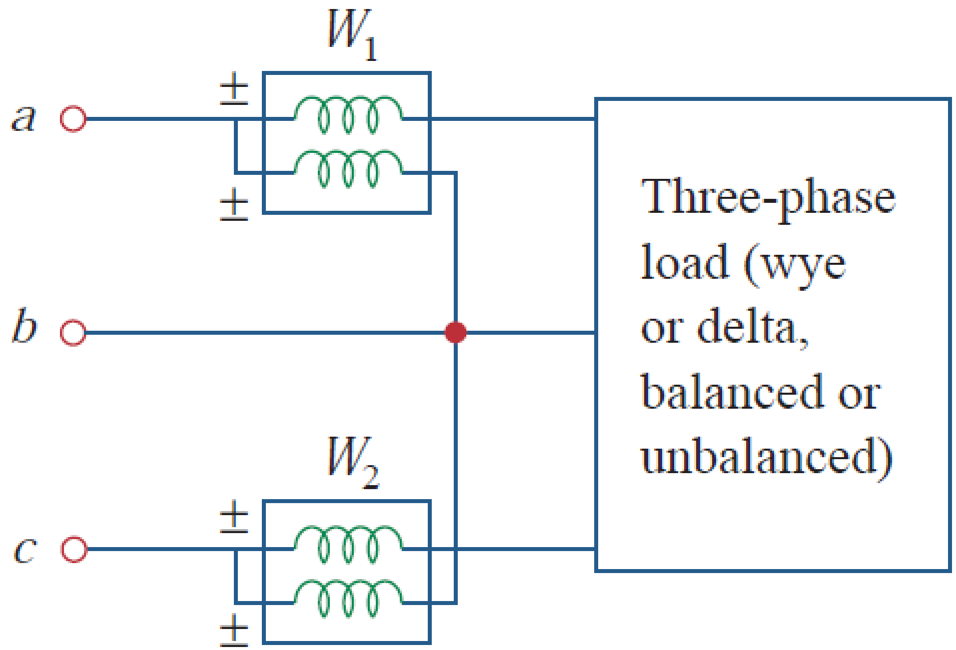


Figure 12.34
Two-wattmeter method for measuring three-phase power.

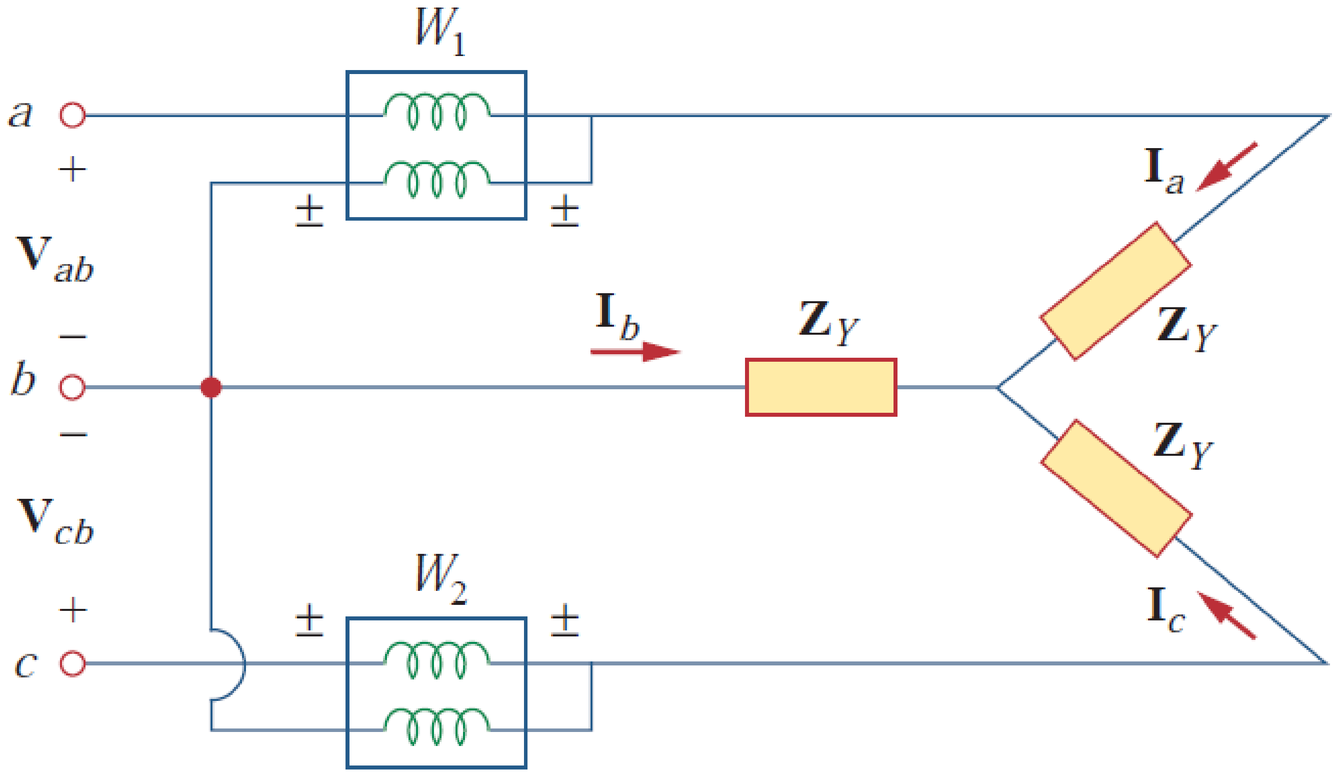


Figure 12.35

Two-wattmeter method applied to a balanced wye load.

$$P_1 = \mathrm{Re}[\mathbf{V}_{ab}\mathbf{I}_a^*] = V_{ab}I_a\cos(\theta + 30^\circ) = V_L I_L \cos(\theta + 30^\circ)$$

$$P_2 = \mathrm{Re}[\mathbf{V}_{cb}\mathbf{I}_c^*] = V_{cb}I_c\cos(\theta - 30^\circ) = V_L I_L \cos(\theta - 30^\circ)$$

$$
\begin{aligned}
P_1 + P_2 &= V_L I_L[\cos(\theta + 30^\circ) + \cos(\theta - 30^\circ)] \\
&= V_L I_L(\cos\theta \cos 30^\circ - \sin\theta \sin 30^\circ \\
&\qquad + \cos\theta \cos 30^\circ + \sin\theta \sin 30^\circ) \\
&= V_L I_L 2 \cos 30^\circ \cos\theta = \sqrt{3} V_L I_L \cos\theta
\end{aligned}
$$

$$\boxed{P_T = P_1 + P_2}$$

$$
\begin{aligned}
P_1 - P_2 &= V_L I_L[\cos(\theta + 30°) - \cos(\theta - 30°)] \\
&= V_l I_L(\cos\theta \cos 30° - \sin\theta \sin 30° \\
&\qquad - \cos\theta \cos 30° - \sin\theta \sin 30°) \qquad \textbf{(12.68)} \\
&= -V_L I_L 2 \sin 30° \sin\theta \\
P_2 - P_1 &= V_L I_L \sin\theta
\end{aligned}
$$

since $2 \sin 30° = 1$. Comparing Eq. (12.68) with Eq. (12.51) shows that the difference of the wattmeter readings is proportional to the total reactive power, or

$$\boxed{Q_T = \sqrt{3}(P_2 - P_1)} \qquad \textbf{(12.69)}$$

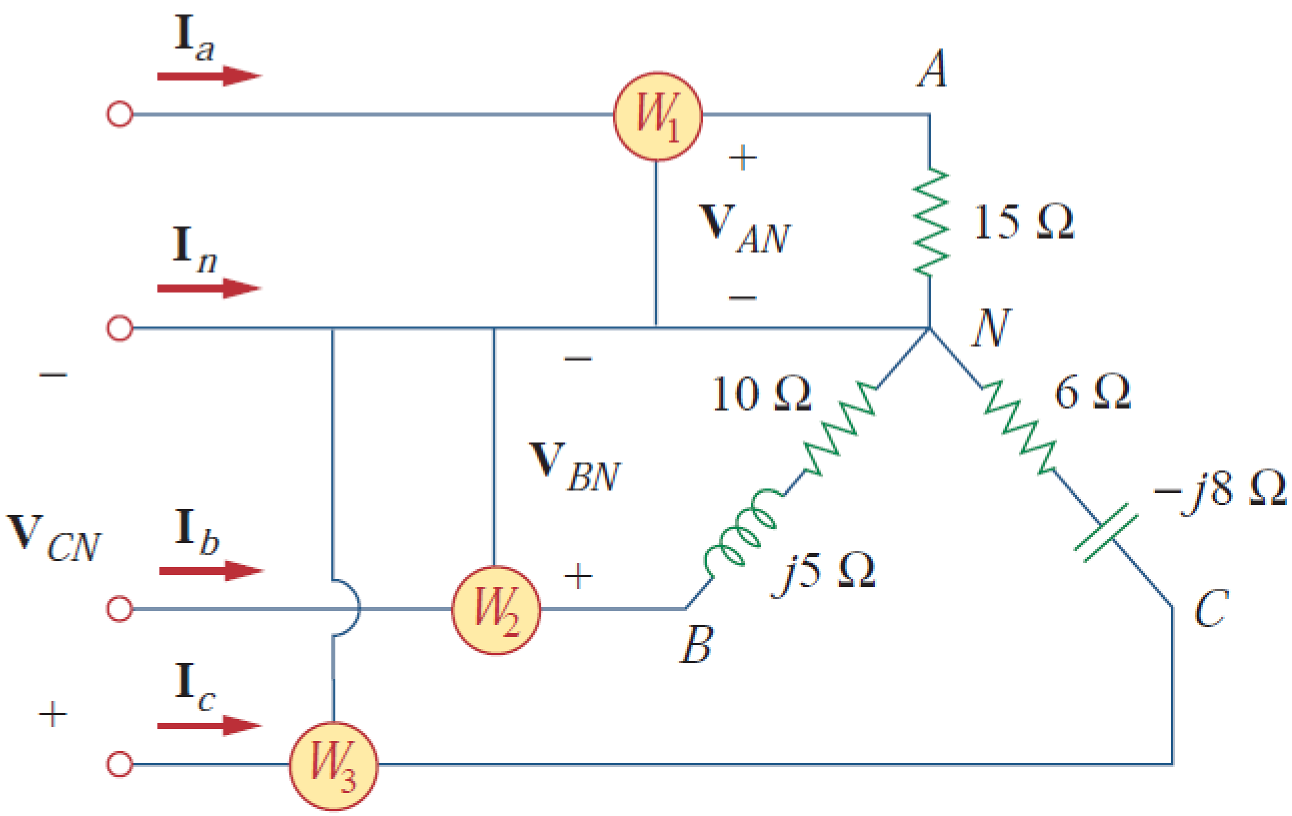


Figure 12.36
For Example 12.13.

Residential wiring in USA

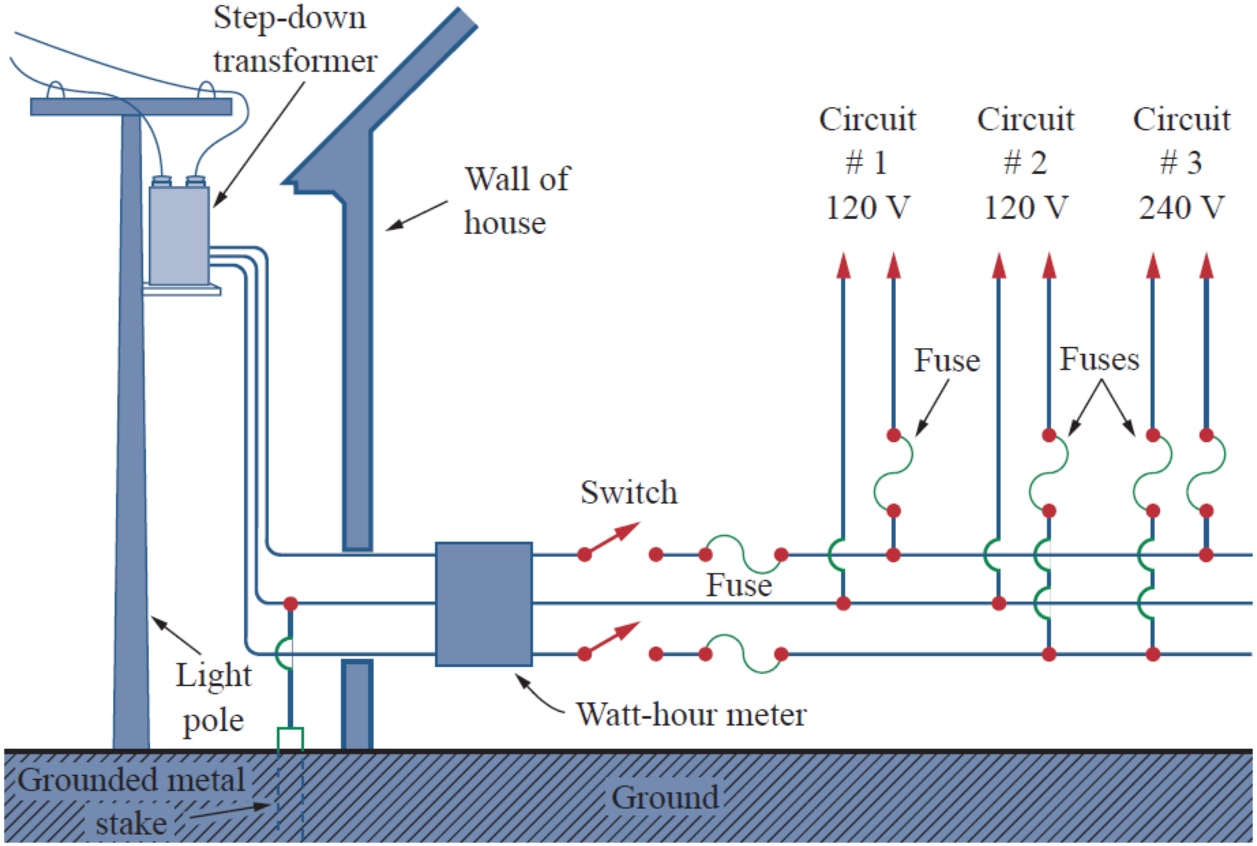


Figure 12.37
A 120/240 household power system.

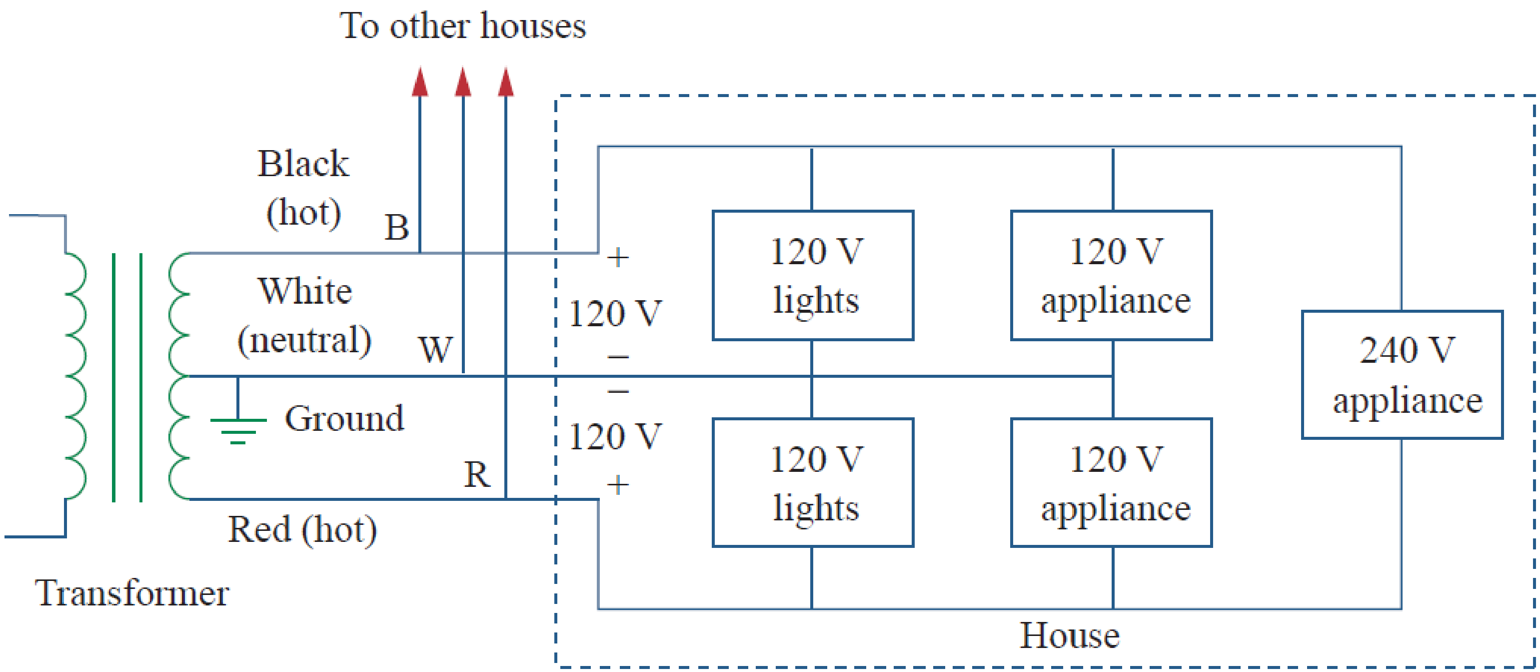


Figure 12.38
Single-phase three-wire residential wiring.