Chapter 18
Two-Port Networks

A **two-port network** is an electrical network with two separate ports for input and output.

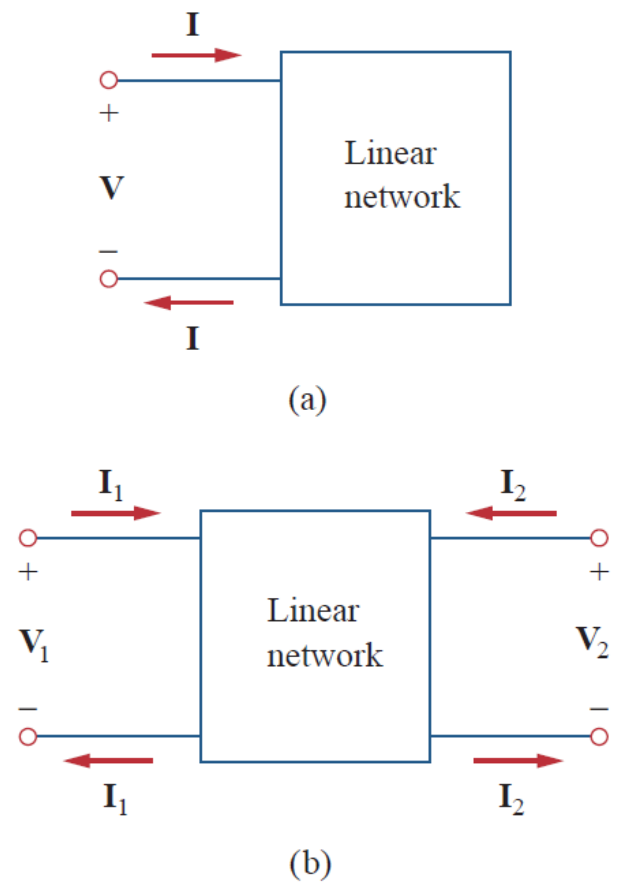


Figure 19.1
(a) One-port network, (b) two-port network.

Impedance Parameters

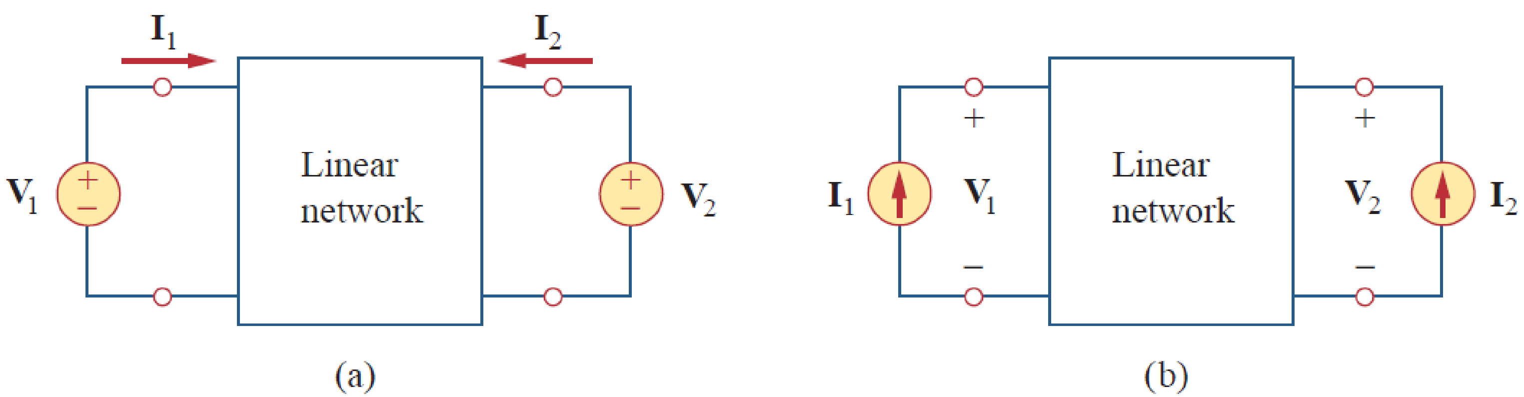
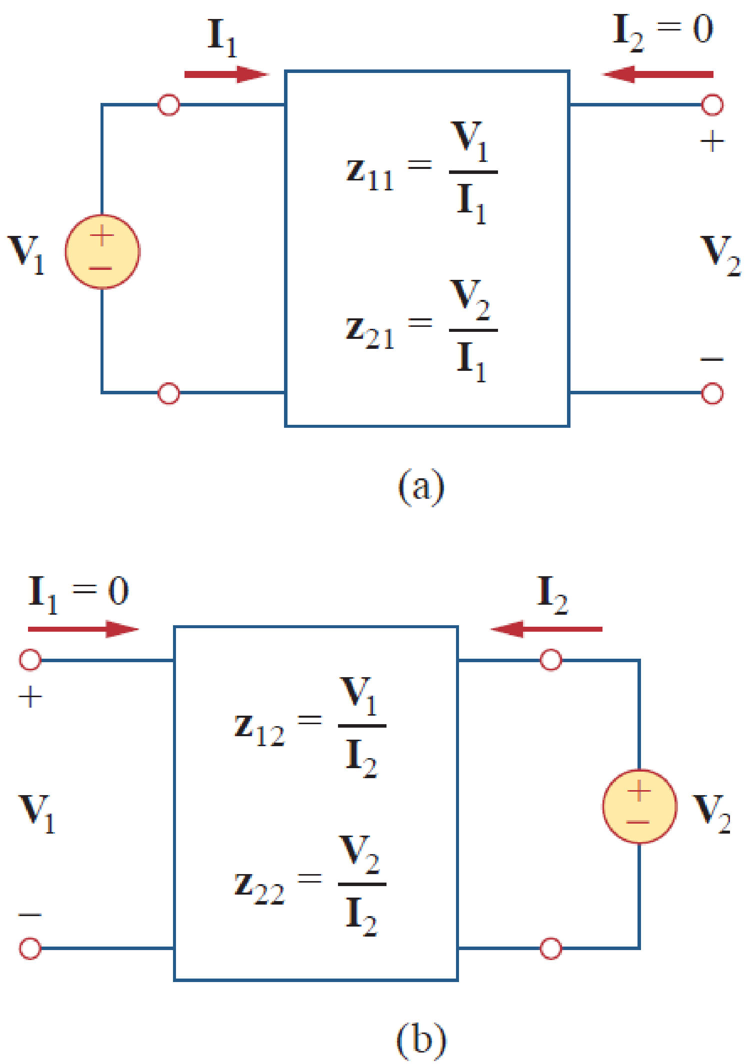


Figure 19.2
The linear two-port network: (a) driven by voltage sources, (b) driven by current sources.

$$\boxed{\begin{aligned} \mathbf{V}_1 &= \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{aligned}}$$

or in matrix form as

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$



$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}, \qquad z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

$$z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}, \qquad z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

Figure 19.3
Determination of the *z* parameters:
(a) finding $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, (b) finding $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$.

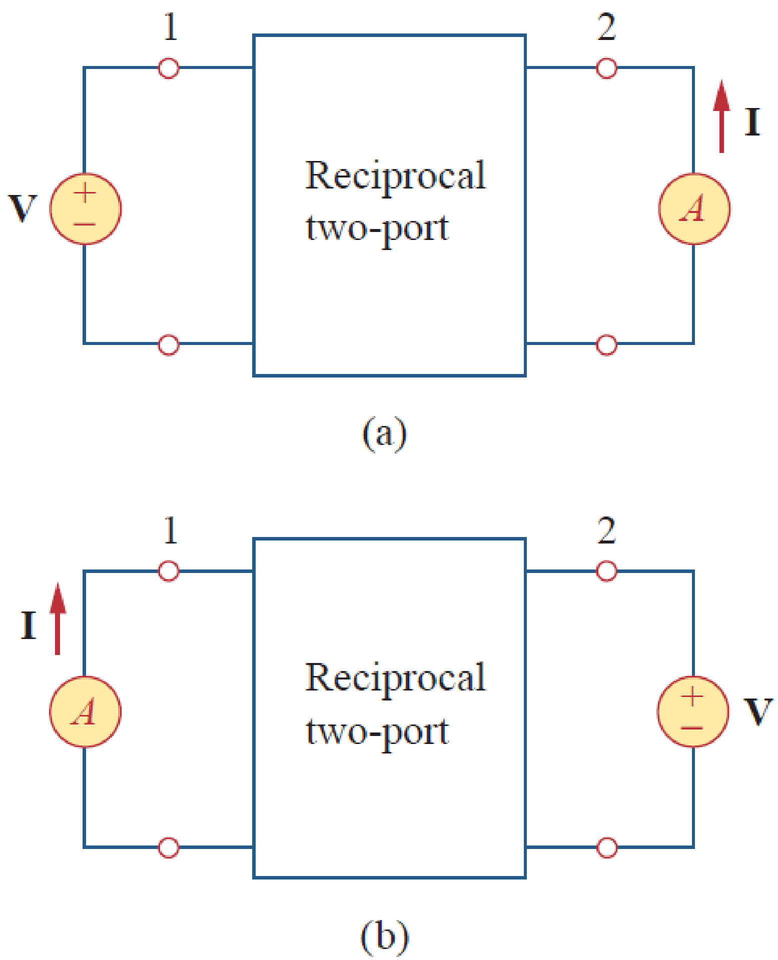


Figure 19.4
Interchanging a voltage source at one port with an ideal ammeter at the other port produces the same reading in a reciprocal two-port.

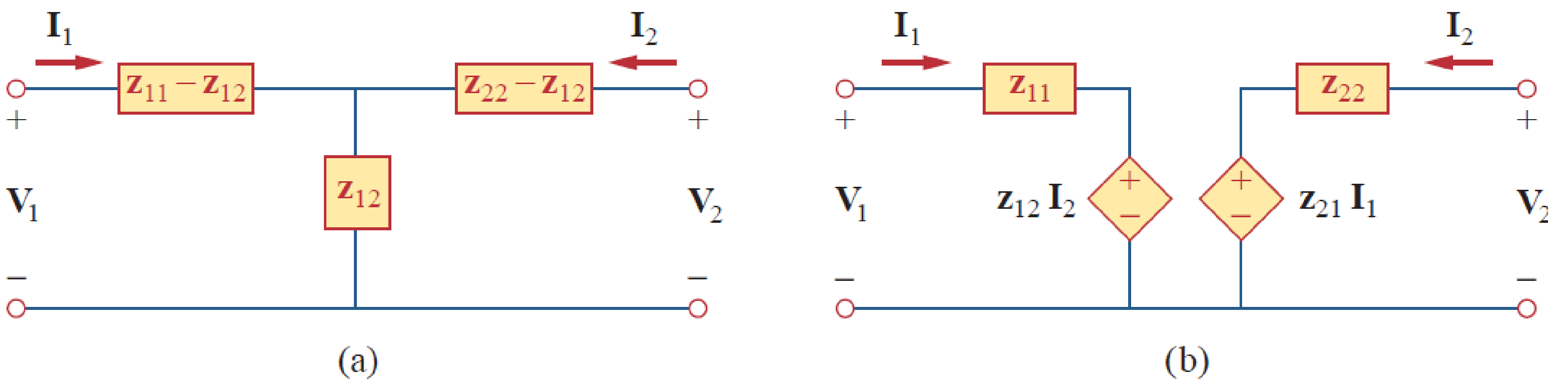


Figure 19.5
(a) T-equivalent circuit (for reciprocal case only), (b) general equivalent circuit.

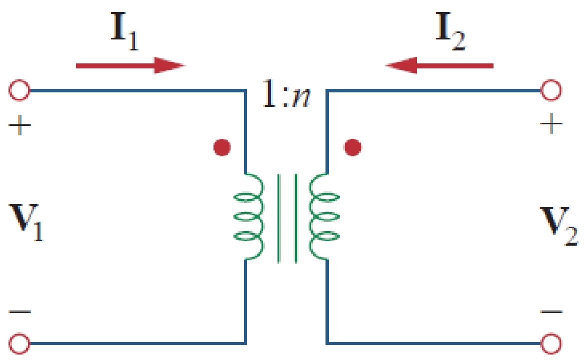


Figure 19.6
An ideal transformer has no *z* parameters.

$$\mathbf{V}_1 = \frac{1}{n}\mathbf{V}_2, \qquad \mathbf{I}_1 = -n\mathbf{I}_2$$

Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit in Fig. 19.10.

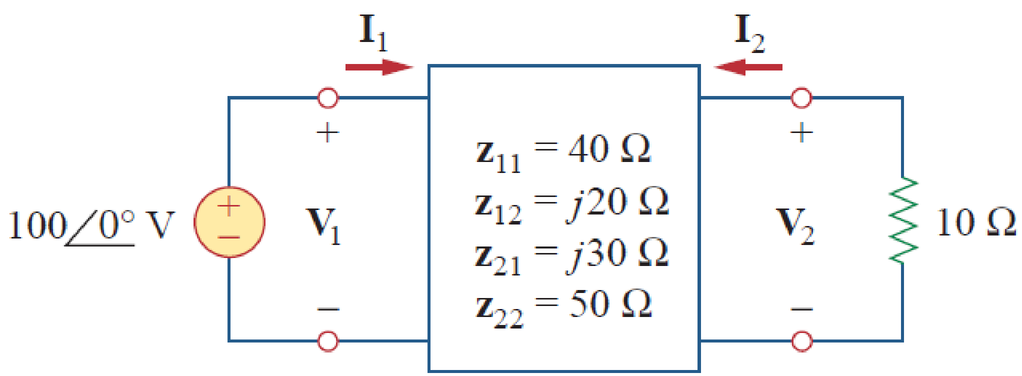


Figure 19.10
For Example 19.2.

ADMITTANCE PARAMETERS

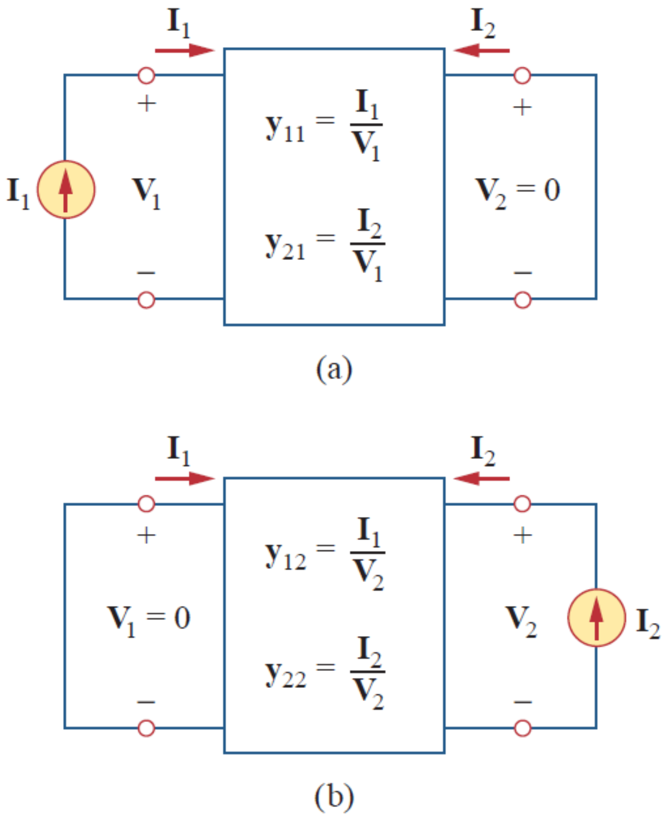


Figure 19.12
Determination of the y parameters: (a) finding y_{11} and y_{21}, (b) finding y_{12} and y_{22}.

$$\boxed{\begin{aligned} \mathbf{I}_1 &= \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \end{aligned}}$$

matrix form as

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{y}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

The values of the parameters can be determined by setting $\mathbf{V}_1 = 0$ (input port short-circuited) or $\mathbf{V}_2 = 0$ (output port short-circuited). Thus,

$$\boxed{\begin{aligned} \mathbf{y}_{11} &= \frac{\mathbf{I}_1}{\mathbf{V}_1}\bigg|_{\mathbf{V}_2=0}, \qquad & \mathbf{y}_{12} &= \frac{\mathbf{I}_1}{\mathbf{V}_2}\bigg|_{\mathbf{V}_1=0} \\ \mathbf{y}_{21} &= \frac{\mathbf{I}_2}{\mathbf{V}_1}\bigg|_{\mathbf{V}_2=0}, \qquad & \mathbf{y}_{22} &= \frac{\mathbf{I}_2}{\mathbf{V}_2}\bigg|_{\mathbf{V}_1=0} \end{aligned}} \tag{19.10}$$

Since the *y* parameters are obtained by short-circuiting the input or output port, they are also called the *short-circuit admittance parameters*. Specifically,

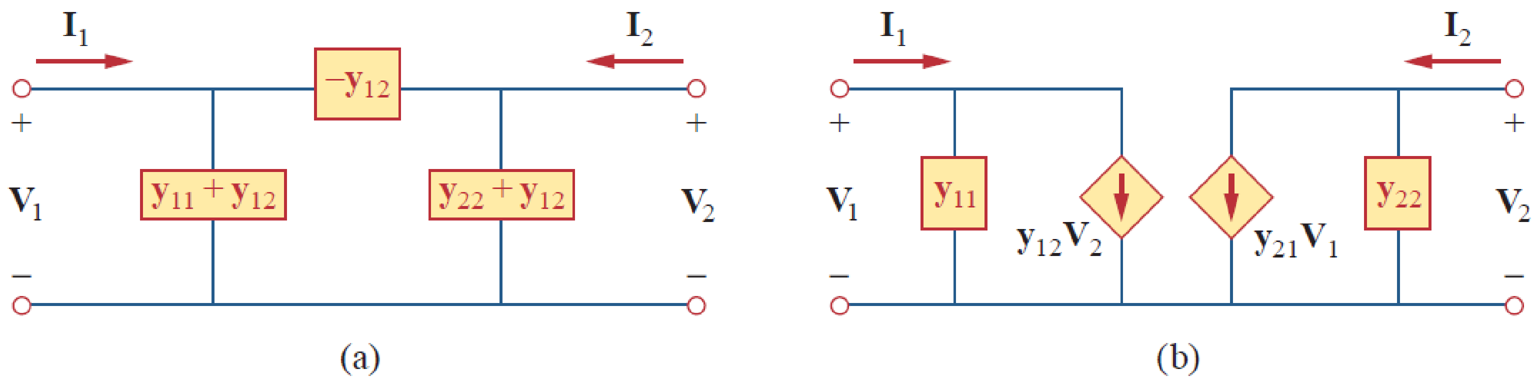


Figure 19.13
(a) Π-equivalent circuit (for reciprocal case only), (b) general equivalent circuit.

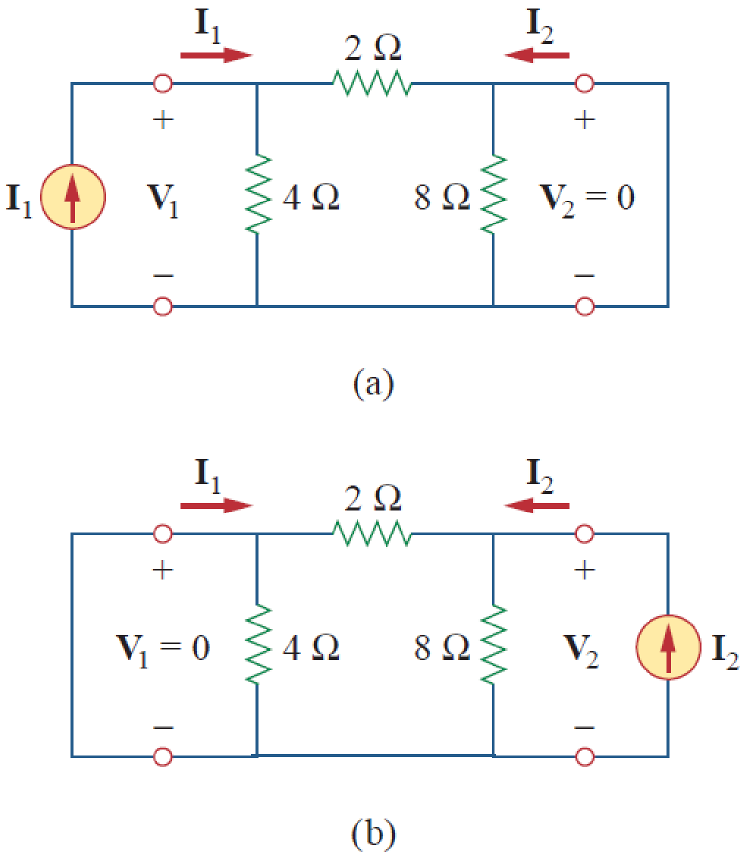


Figure 19.15
For Example 19.3: (a) finding $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, (b) finding $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$.

Example 19.4

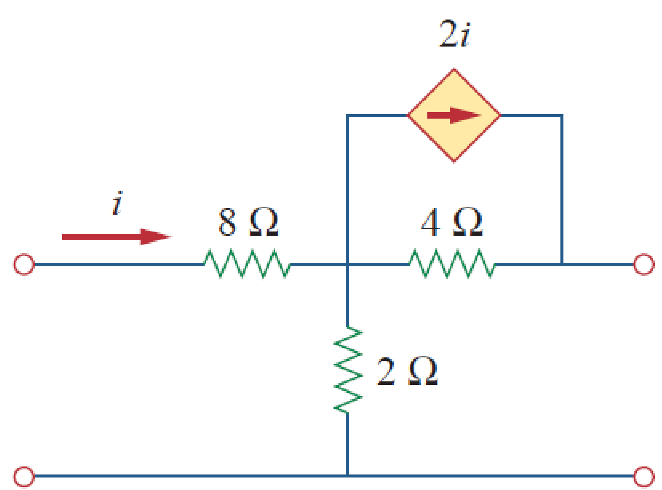


Figure 19.17
For Example 19.4.

Example 19.4

l1
t-

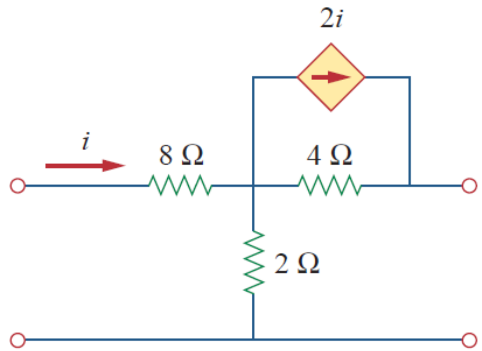


Figure 19.17
For Example 19.4.

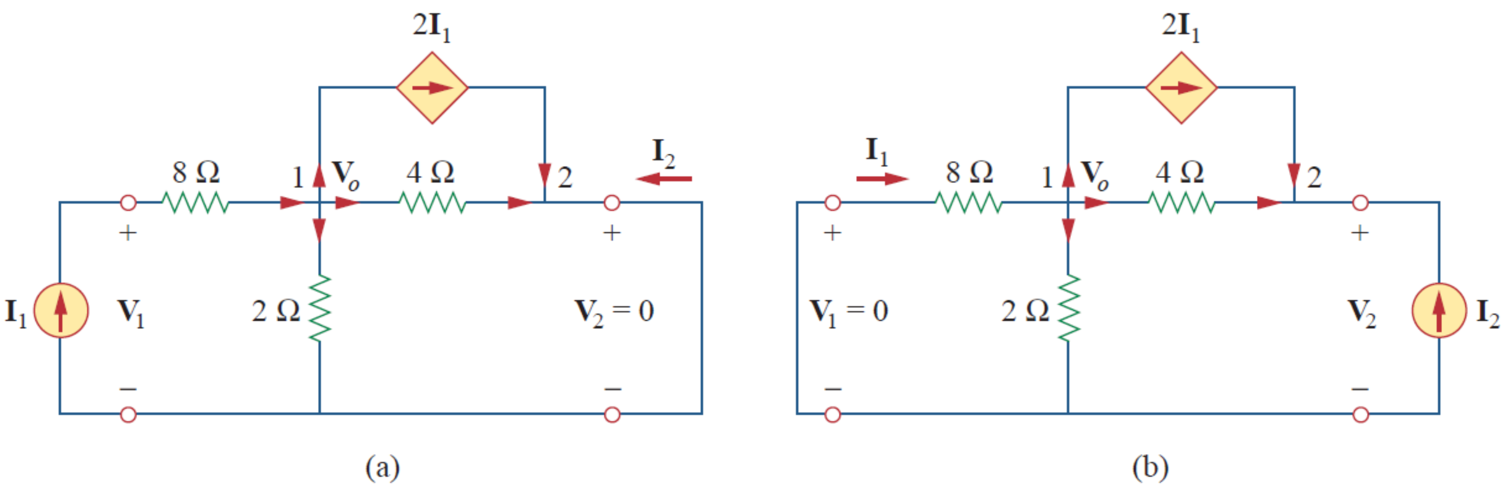


Figure 19.18
Solution of Example 19.4: (a) finding $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, (b) finding $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$.

Hybrid Parameters

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

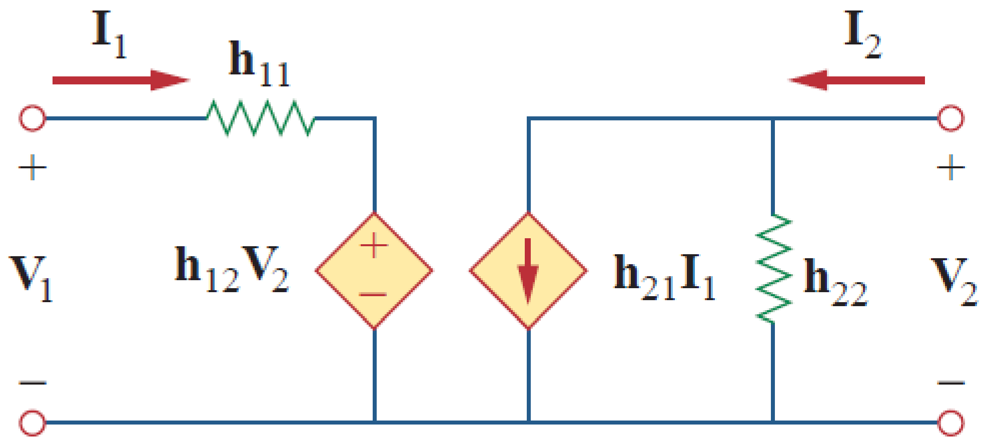


Figure 19.20

The *h*-parameter equivalent network of a two-port network.

A set of parameters closely related to the h parameters are the g *parameters* or *inverse hybrid parameters*. These are used to describe the terminal currents and voltages as

$$\boxed{\begin{aligned} \mathbf{I}_1 &= \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2 \end{aligned}} \qquad \textbf{(19.18)}$$

or

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{11} & \mathbf{g}_{12} \\ \mathbf{g}_{21} & \mathbf{g}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{g}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} \qquad \textbf{(19.19)}$$

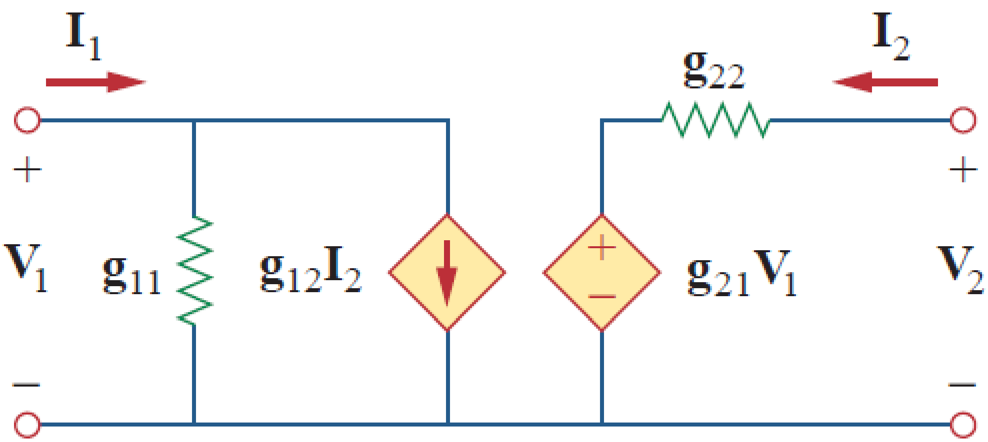


Figure 19.21

The *g*-parameter model of a two-port network.

Determine the Thevenin equivalent at the output port of the circuit in Fig. 19.25.

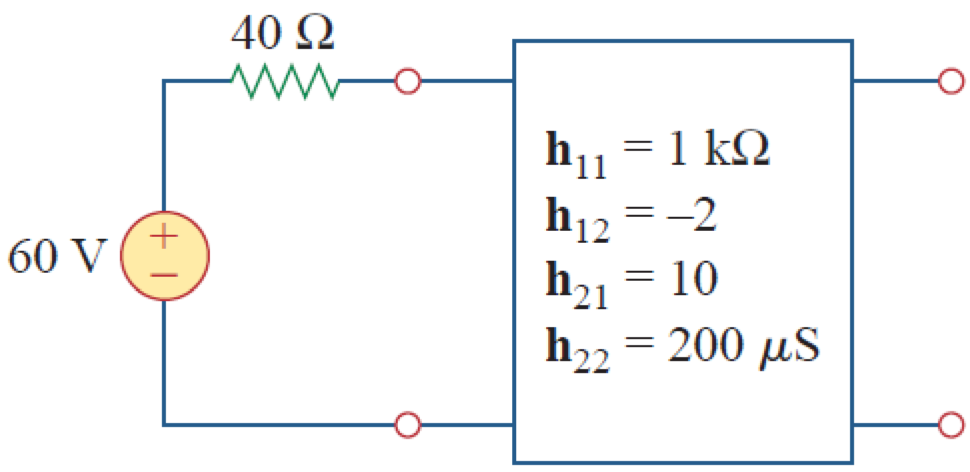


Figure 19.25
For Example 19.6.

Transmission Parameters

Another set of parameters relates the variables at the input port to those at the output port. Thus,

$$\boxed{\begin{aligned} \mathbf{V}_1 &= \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2 \\ \mathbf{I}_1 &= \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2 \end{aligned}} \tag{19.22}$$

or

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix} = [\mathbf{T}] \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix} \tag{19.23}$$

The transmission parameters are determined as

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2}\bigg|_{\mathbf{I}_2=0}, \qquad \mathbf{B} = -\frac{\mathbf{V}_1}{\mathbf{I}_2}\bigg|_{\mathbf{V}_2=0}$$

$$\mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2}\bigg|_{\mathbf{I}_2=0}, \qquad \mathbf{D} = -\frac{\mathbf{I}_1}{\mathbf{I}_2}\bigg|_{\mathbf{V}_2=0}$$

Thus, the transmission parameters are called, specifically,

A = Open-circuit voltage ratio

B = Negative short-circuit transfer impedance

C = Open-circuit transfer admittance

D = Negative short-circuit current ratio

$$\begin{aligned} \mathbf{V}_2 &= \mathbf{a}\mathbf{V}_1 - \mathbf{b}\mathbf{I}_1 \\ \mathbf{I}_2 &= \mathbf{c}\mathbf{V}_1 - \mathbf{d}\mathbf{I}_1 \end{aligned}$$

or

$$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ -\mathbf{I}_1 \end{bmatrix} = [\mathbf{t}] \begin{bmatrix} \mathbf{V}_1 \\ -\mathbf{I}_1 \end{bmatrix} \tag{19.27}$$

The parameters **a**, **b**, **c**, and **d** are called the *inverse transmission*, or t, *parameters.* They are determined as follows:

$$\begin{aligned} \mathbf{a} &= \left.\frac{\mathbf{V}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_1=0}, & \mathbf{b} &= -\left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_1=0} \\ \mathbf{c} &= \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_1=0}, & \mathbf{d} &= -\left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_1=0} \end{aligned} \tag{19.28}$$

In terms of the transmission or inverse transmission parameters, a network is reciprocal if

$$\boxed{\mathbf{AD} - \mathbf{BC} = 1, \qquad \mathbf{ad} - \mathbf{bc} = 1} \qquad \textbf{(19.30)}$$

These relations can be proved in the same way as the transfer impedance relations for the z parameters. Alternatively, we will be able to use Table 19.1 a little later to derive Eq. (19.30) from the fact that $\mathbf{z}_{12} = \mathbf{z}_{21}$ for reciprocal networks.

The **ABCD** parameters of the two-port network in Fig. 19.34 are

$$\begin{bmatrix} 4 & 20\ \Omega \\ 0.1\ \text{S} & 2 \end{bmatrix}$$

The output port is connected to a variable load for maximum power transfer. Find R_L and the maximum power transferred.

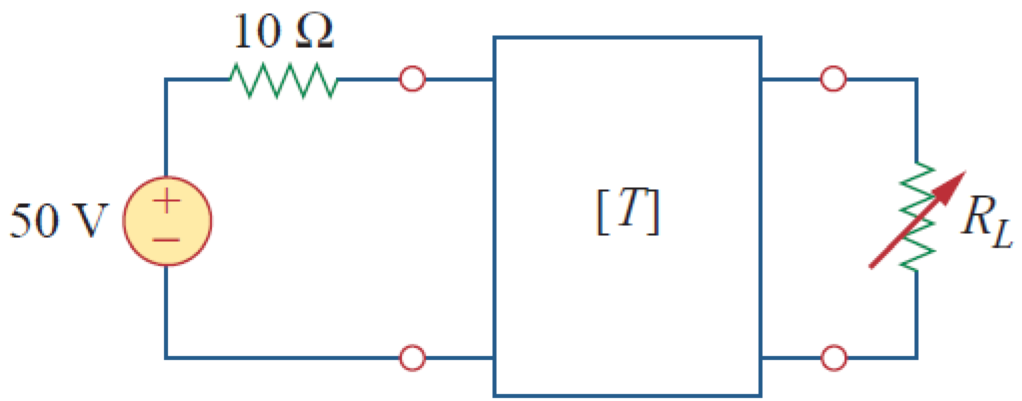


Figure 19.34
For Example 19.9.

Interconnection of Networks

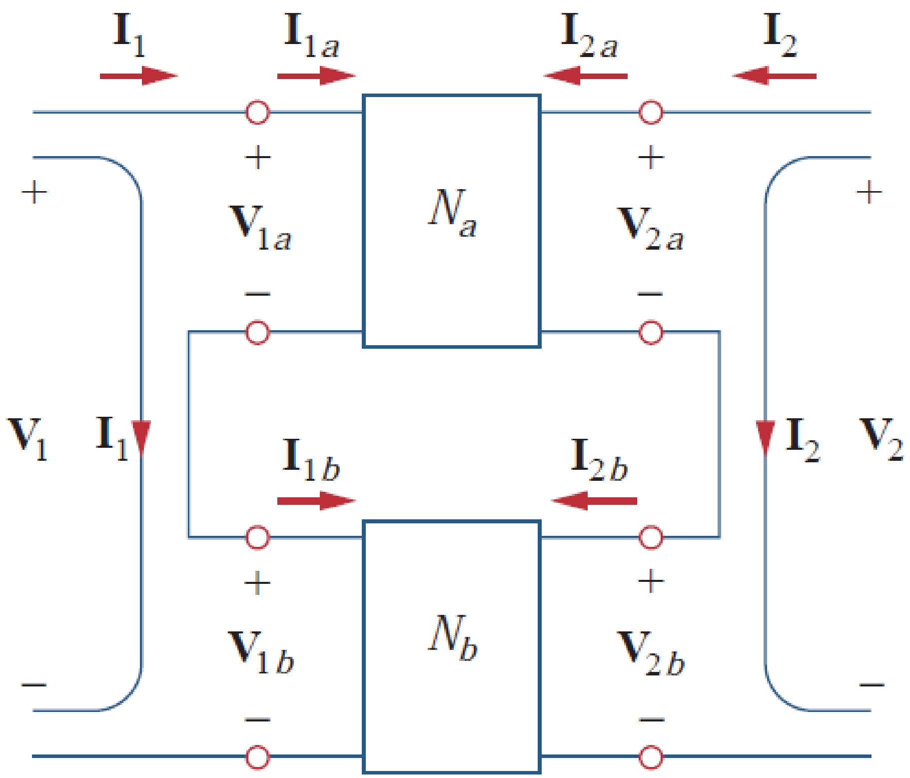


Figure 19.39
Series connection of two two-port networks.

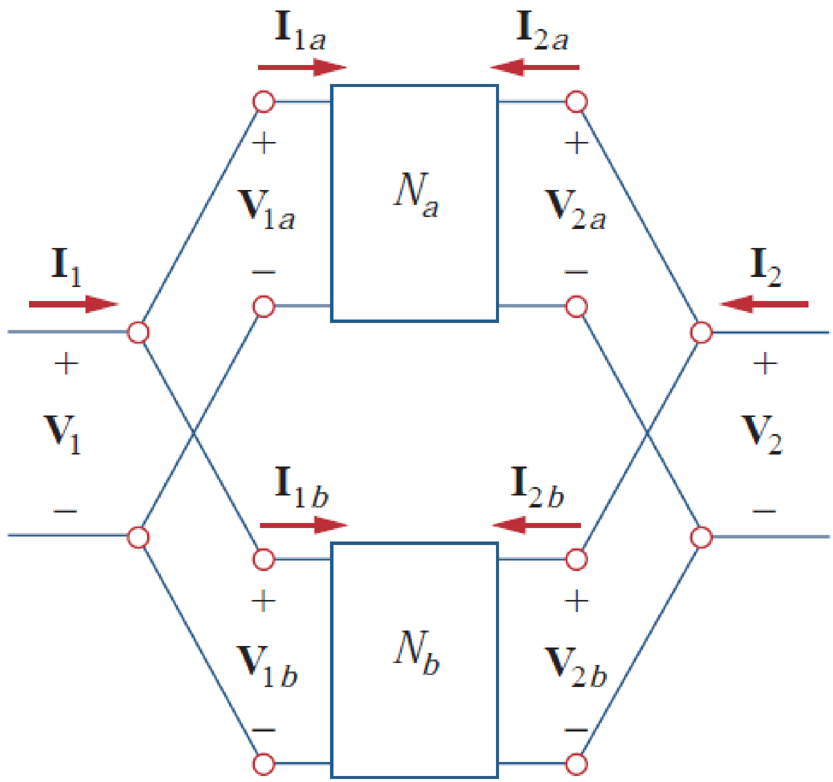


Figure 19.40
Parallel connection of two two-port networks.

$$\boxed{[\mathbf{T}] = [\mathbf{T}_a][\mathbf{T}_b]} \tag{19.61}$$

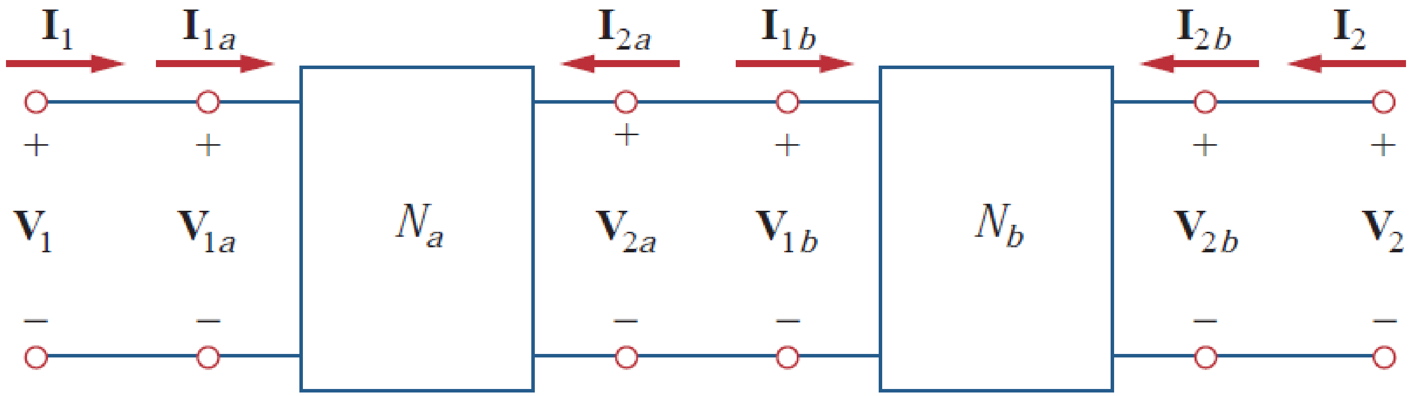


Figure 19.41

Cascade connection of two two-port networks.

Evaluate $\mathbf{V}_2/\mathbf{V}_s$ in the circuit in Fig. 19.42.

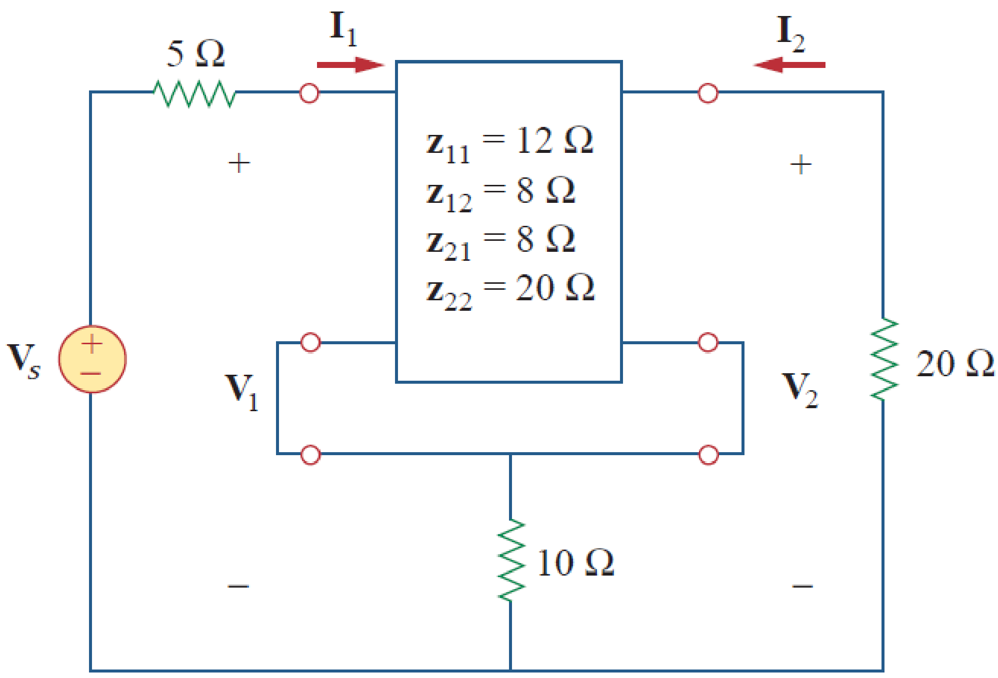


Figure 19.42
For Example 19.12.

Find the y parameters of the two-port in Fig. 19.44.

Example 19.13

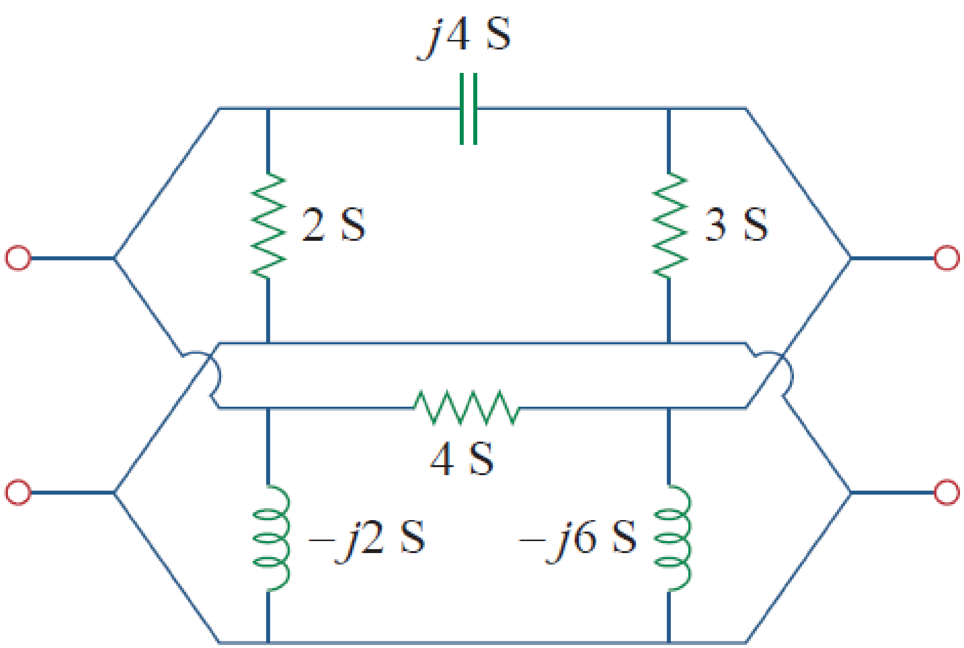


Figure 19.44
For Example 19.13.